On a Refinement-Free Strategy for Preconditioning Electromagnetic Integral Equations in the High Frequency Regime

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Electromagnetic scattering by prefect electrical conductors (PECs) is very often modeled with integral equation techniques solved with the Boundary Element Method (BEM). The popularity of this approach is due to the fact that BEM schemes only discretize material boundaries and automatically impose radiation conditions. Moreover, the use of fast techniques reduces the complexity of the matrix-vector product to linear despite the dense nature of the interaction matrices.

Most PEC integral formulations do however suffer from conditioning issues, which reduce the convergence rate of the iterative solvers and thus prevent the overall linearity of the solution process. The condition number increase has been widely studied (i) in the low frequency regime in which the discretization of the scatter is kept constant while the frequency decreases and (ii) in the dense discretization regime in which the frequency is kept constant and the mesh is refined. However, the condition number also grows unbounded when (iii) the number of unknowns per wavelength is kept constant with increasing frequency.

The electric field integral equation (EFIE) has been the focus of numerous investigations aimed at addressing issues (i) and (ii). The problem of the low-frequency breakdown is often tackled with Helmholtz decomposition techniques such as loop star/loop tree decompositions or via the recovery of auxiliary quantities. Both approaches have limitations: the former exacerbates problem (ii) while the latter is computationally burdensome. The techniques presented for solving the dense discretization breakdown (ii) often rely on the well-known Calderón identities to precondition the electric field integral operator (EFIO) with itself. More recently, a formulation immune to both (i) and (ii) has been proposed proposed in (Andriulli, F. P. et al. IEEE transactions on antennas and propagation, 61(4), 2077-2087,2013) and relies on the usage of Calderón identities coupled with quasi-Helmholtz projectors to stabilize the first two problems. A barycentric refinement free version of the Calderón approach was presented in (Adrian, S.B. et al. Journal of Computational Physics 376 (2019): 1232–52. These schemes, however, are not capable capable of addressing the high-frequency breakdown.

In this work a new approach leveraging on quasi-Helmholtz projectors and Helmholtz equations to tackle problem (iii) while still addressing (i) and (ii) is introduced. A quasi-Helmholtz projector analysis will allow for a separate regularization of both low and high frequency components of the spectrum, while the elliptic part of the operator will still be regularized in a Calderón-like way. This talk will focus on both theoretical developments and on numerical results which will corroborate the theory and show the impact of this new computational strategy.