On the Use of Machine Learning Strategies in Preconditioning Electromagnetic Integral Equations

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Integral equations are widely used in computational electromagnetics to solve radiation and scattering problems. Their efficiency resides in the fact that they require only a surface discretization and automatically impose radiation conditions. Among integral formulations, the Electric Field Integral Equation (EFIE) is one of the most prominent, often used in combination with the Magnetic Field Integral Equation (MFIE), giving rise to the Combined Field Integral Equation (CFIE), which is immune from spurious internal resonances.

Both the EFIE and CFIE, however, suffer from severe ill-conditioning problems when the discretization density is high, the frequency is low, or in the high frequency regime. Ill conditioned systems give rise to an high number of iterations when employing iterative solvers and to numerical instabilities for both iterative and direct solutions and are thus quite undesirable to be solved. In addition, severe ill conditioning of the system limits the efficiency of fast solvers that could otherwise reduce the computational complexity of the problem to quasi-linear by allowing for fast matrix-vector multiplication.

Effective Preconditioning strategies are often based on the grasping of the main spectral properties of the operator to be regularized. This could occur via the choice of proper basis functions or via the use of semi-analytic regularization (like Calderón strategies). One limitation of these purely theoretical approaches is that some key results or optimal parameters are only available for canonical and simple structures, typically spheres. Thus, while still effective on more general structures, some of these approaches could greatly benefit from a geometry-dependent tailoring. Yet, even on simple structures, the amount of work required would be intractable and unpractical.

In this work we will explore a different approach based on Machine Learning. The key spectral features of the original operator are tracked by a learning process and then pipelined to build an optimal or quasi-optimal preconditioner.

Proper priors on the preconditioning structure, arising by standard preconditioning strategies will be harmonized with the learned elements to give rise to a new family of regularizing schemes which will address several regimes of interest for the EFIE and related equations.

Theoretical developments will be alternated with numerical results which will corroborate the theory and will show the practical impact of our new methods.