Construction and Application of Geometrically Optimal Curvilinear Surface Elements for Double Higher-Order MoM-SIE Modeling

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This paper presents the construction and application of automated curvilinear quadrilateral mesh generation for higher-order method of moments (MoM) in the surface integral equation (SIE) formulation. In particular, we apply the developed mesh generation and reconstruction (remeshing) algorithms to double (geometrical and current) higher-order (DHO) MoM-SIE modeling technique (M. Djordjevic and B. M. Notaros, IEEE Transactions on Antennas and Propagation, Vol. 52, 2118-2129, 2004), using generalized curvilinear quadrilateral patches of arbitrary geometrical orders for geometrical modeling and hierarchical divergence-conforming polynomial vector basis functions of arbitrary orders for current modeling. The DHO MoM-SIE method uses a singularity extraction technique to deal with singular or near-singular integrals, arising for zero or small source-to-field distances, in the MoM analysis, consisting of analytical integration of a principal singular part of the integrand over a (generally not rectangular) parallelogram whose surface is close to the surface of the generalized quadrilateral near the singular point and numerical integration of the rest using Gauss–Legendre quadrature formulas.

A primary ingredient for optimal higher order solutions, the accuracy and quality of the geometrical structure under analysis, requires significant consideration. Creating, however, a geometrical discretization which takes full advantage of DHO methods poses a difficult challenge. Moreover, when the corner angles in the patches which compose a structure exceed ~120 degrees, error in the solution increases, due to a reduction in accuracy and/or efficiency in the combined numerical/analytical integration technique based on the singularity extraction, dictating the need for stricter conditions on the quality of mesh reconstructions. Advanced remeshing techniques, therefore, provide ample potential in terms of increasing accuracy and efficiency, especially for higher order methods with curved elements (DHO modeling).

We discuss first the requisite steps to transform a triangular surface mesh into one of generalized quadrilaterals utilizing conformal parameterization methods. Conformal parameterization, typically seen in the field of computer graphics for mesh texturing, helps achieve quality mesh constructions through its angle preserving, rather than area preserving, properties. Further, with a precomputed parameterization, remeshing for higher or lower resolutions incurs a relatively low additional cost. We then demonstrate the quality of the reconstructions through an examination of corner angles. Additionally, we introduce adaptive sampling techniques which help preserve sharp geometries more efficiently compared to uniform sampling, and present considerations of electrical size for the elements in the reconstructions.

Applying the reconstructions to scattering simulations in the DHO MoM-SIE, we demonstrate the utility and advantage of higher-order elements constructed using our method. Specifically, we discuss computational efficiency and accuracy for several scattering examples, including both lower and higher order elements, illustrating the ability of curvilinear quadrilateral patches to reduce the number of unknowns in the MoM-SIE system, while preserving accuracy with minimal user input.