Singular Integration by Interpolation (SIBI) for Integral Equations: A Tensor Decomposition Approach

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The integration of singular (often referred to as hyper-singular), nearly singular , weakly-singular (often referred to as singular) and nearly weakly-singular interactions in the Boundary Element Method (BEM) solution of the Electric (EFIE) and Magnetic Field Integral equation (MFIE) is ever so critical to the accuracy of near-field engineering quantiles (e.g. input impedance) matrix conditioning and speed of matrix fill. Considerable amount of research has been devoted to this topic, with the most prominent techniques being singularity subtraction (Järvenpää, et al, IEEE Trans. on Antennas Propagat., 54, 42-49, 2006), singularity cancellation (Polimeridis and Mosig, IEEE Trans. on Antennas Propagat., 58, 1980-1988, 2010) and full 4D integral evaluation (Wilton, et al., IEEE Trans. on Antennas Propagat., 65, 2479-2493, 2017). To produce respectable accuracies, say 4 significant digits, more or less all those methods require numerous and often complicated function evaluations, that adversely affect the overall speed.

The method of Singular Integration by Interpolation (SIBI) was recently proposed in (Kyriakou, et. al., IEEE APS, 2018) to tackle this very speed versus accuracy compromise with a very unconventional approach. Unlike other methods, the singular integration problem is recast into a multidimensional interpolation problem where every singular triangle to triangle interaction, e.g. common triangle or common edge, corresponds to a three or six dimensional function, respectively, which must be interpolated by a set of pre-computed universal (frequency, materials independent) look-up tables. For common triangle interactions this approach was found to be blazingly fast and exceptionally accurate by using sparse grids multidimensional interpolation (Klimke, PhD, Stuttgart Univ., 2006). But, when it came to common edge interactions, although the speed remained very good, it lacked in accuracy. This work will propose an alternative approach that borrows from big-data and artificial intelligence disciplines. More specifically, the high-dimensional interpolation problem will be considered as a tensor (generalization of a dense matrix in more than two dimensions) and application of various low-rank tensor decompositions to save memory and speed-up computation will be attempted. Low rank Tensor Tucker (Kolda and Bader, SIAM, 51, 3, 455-500, 2009) or Tensor Train (Oseledets and Tyrtyshnikov, Linear Algebra and Applications, 432, 70-88, 2010) decompositions will be considered, and will be compared to sparse grids in the context of SIBI. The accuracy and speed of this SIBI tensor decomposition approach in comparison to state-of-the-art for various realistic PEC triangular RWG meshes for integral equations will be presented.