A Platform Green's Function Method for In-Situ Antenna Analysis and Design

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***Abstract*—The objective of this paper is to build a reconfigurable, reusable, and parallel model reduction platform towards transformative in-situ antenna design. The key idea is to introduce a separable and compressible platform Green's function in an up-front offline computation. Once obtained, the online computational complexity does not depend on the size of the in-situ platform. As a result, in-situ design and optimization of multi-antenna systems can be performed at the same cost as the free-space radiation. The advancements make high-fidelity in-situ antenna design orders of magnitude faster.**

***Index Terms*—antenna, domain decomposition, Green's function, integral equation, model reduction.**

I. OVERVIEW

Modern military and commercial EM systems are routinely equipped with multiple antennas serving for radar and wireless communications. The computational electromagnetics (CEM) has emerged as a powerful and indispensable tool to evaluate the in-situ performance and co-site interference. These simulations are enabled by fast and rigorous numerical solutions of Maxwell's Equations as well as rapid advances in high-performance computing (HPC) systems [1], [2]. Nevertheless, the CEM based in-situ antenna design goes beyond just performing a single simulation. It often needs to perform a number of simulations in order to navigate highly complex design spaces. Each simulation should complete within at most a few minutes even a few seconds in an industrial design environment. Clearly, fundamental research into innovative mathematics and algorithms are required.

This work aims to address this challenging engineering need. Key ingredients are summarized as follows: **(1)** The discontinuous Galerkin (DG) boundary element (BE) method and geometry-aware domain decomposition (DD) method [3] are employed to facilitate a modular design-oriented decomposition. **(2)** A novel platform Green's function (PGF) is introduced on the outer surface of those antennas. The PGF is calculated once in the offline phase to characterize the coupling with large, fixed platform. It can be reused for all future in-situ computation. **(3)** In the online phase, rapid solution for multi-query antenna design needs is achieved by a Schwarz DD solver of the reduced order system. The computational costs are the same as the free-space radiation.

II. METHODOLOGY

(1) Problem decomposition: Consider the in-situ analysis of two antennas on the high-definition platform, as illustrated in Fig. 1. The problem can be decomposed into three sub-regions: sub-regions Ω_1 and Ω_2 contain two antennas, sub-region Ω_3 is electrically large PEC platform. We have used surfaces, Γ_{31} and Γ_{32}, to facilitate the decomposition. The size of the surfaces is determined by the prescribed surface area where the antennas are allowed to be mounted.

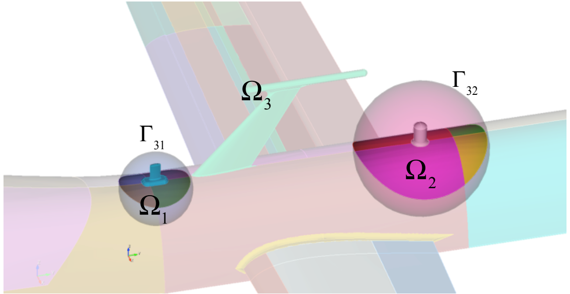


Fig. 1: DD based model reduction for in-situ antenna analysis.

The finite element (FE) method is used to discretized the volume domain of Ω_1 and Ω_2, and the DG boundary element method is applied to the exterior surface of Ω_1 and Ω_2 and platform Ω_3. The resulting system matrix can be written as:

$$\begin{bmatrix} \begin{bmatrix} \mathcal{A}_1^{\mathrm{FE}} & \mathcal{N}_1 \\ \mathcal{N}_1^T & \mathbf{A}_1^{\mathrm{BE}} \end{bmatrix} & \begin{matrix} \\ \mathbf{C}_{12}^{\mathrm{BE}} \end{matrix} & \begin{matrix} \\ \mathbf{C}_{13}^{\mathrm{BE}} \end{matrix} \\ \begin{matrix} \\ \mathbf{C}_{21}^{\mathrm{BE}} \end{matrix} & \begin{bmatrix} \mathcal{A}_2^{\mathrm{FE}} & \mathcal{N}_2 \\ \mathcal{N}_2^T & \mathbf{A}_2^{\mathrm{BE}} \end{bmatrix} & \begin{matrix} \\ \mathbf{C}_{23}^{\mathrm{BE}} \end{matrix} \\ \mathbf{C}_{31}^{\mathrm{BE}} & \mathbf{C}_{32}^{\mathrm{BE}} & \mathbf{A}_3^{\mathrm{BE}} \end{bmatrix} \tag{1}$$

The resulting sub-domain FE matrices, $\mathcal{A}_m^{\mathrm{FE}}$, m =1,2, have complex nonlinear dependency on a few design parameters (ϵ_m, μ_m, $\mathbf{r}_m$, etc.). The sub-domain BE matrices $\mathrm{A}_m^{\mathrm{BE}}$, m =1,2,3, and coupling matrices $\mathbf{C}_{mn}^{\mathrm{BE}}$ are resulting from free-space Green's function, and has no parametric dependency.

Evidently, the above non-overlapping and non-conforming DG and DD methods lead to a modular design-oriented decomposition. The modification of antenna types, parameters, locations in the design stage is reflected in local and sparse FE matrices, which are decoupled from exterior BE matrices. Nevertheless, every time antenna design is modified, we still need to solve the entire system matrix. That is where the platform Green's function comes into play.

(2) Platform Green's function: The PGF is evaluated on the artificial surfaces, $\Gamma_3 = \Gamma_{31} \cup \Gamma_{32}$, in the offline computing phase. Once calculated, the large fixed platform is rigorously represented by the PGF. The direct calculation of the PGF matrix requires the solution of the large platform Ω_3 with respect to individual unit source currents on Γ_3. Recognizing the coupling between antennas and platform is considerably low-rank, we proposed a novel alternating and random interpolative decomposition (AR-ID) to select the skeleton source currents from the original ones on the exterior surface Γ_3. The AR-ID calculation can be achieved locally per antenna sub-system and embarrassingly in parallel. The reduced model of in-situ antenna system can be written as:

$$\begin{bmatrix} \begin{bmatrix} \mathcal{A}_1^{\mathrm{FE}} & \mathcal{N}_1 \\ \mathcal{N}_1^T & \tilde{\mathbf{A}}_1^{\mathrm{BE}} \end{bmatrix} & \begin{matrix} & \\ & \tilde{\mathbf{C}}_{12}^{\mathrm{BE}} \end{matrix} \\ \begin{matrix} & \\ & \tilde{\mathbf{C}}_{21}^{\mathrm{BE}} \end{matrix} & \begin{bmatrix} \mathcal{A}_2^{\mathrm{FE}} & \mathcal{N}_2 \\ \mathcal{N}_2^T & \tilde{\mathbf{A}}_2^{\mathrm{BE}} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1^{\mathrm{FE}} \\ \mathbf{x}_1^{\mathrm{BE}} \\ \mathbf{x}_2^{\mathrm{FE}} \\ \mathbf{x}_2^{\mathrm{BE}} \end{bmatrix} = \begin{bmatrix} \mathbf{b}_1^{\mathrm{FE}} \\ 0 \\ \mathbf{b}_2^{\mathrm{FE}} \\ 0 \end{bmatrix} \tag{2}$$

The updated BE matrices consist of the free-space GF matrices and the AR-ID representation of PGF matrices:

$$\tilde{\mathbf{A}}_1^{\mathrm{BE}} = \mathbf{A}_1^{\mathrm{BE}} - \mathbf{R}_{13}^{\mathrm{n_1 \times k_1}} \cdot \mathbf{S}_{11}^{\mathrm{k_1 \times k_1}} \cdot \mathbf{R}_{31}^{\mathrm{k_1 \times n_1}} \tag{3}$$

$$\tilde{\mathbf{C}}_{12}^{\mathrm{BE}} = \mathbf{C}_{12}^{\mathrm{BE}} - \underbrace{\mathbf{R}_{13}^{\mathrm{n_1 \times k_1}} \cdot \mathbf{S}_{12}^{\mathrm{k_1 \times k_2}} \cdot \mathbf{R}_{32}^{\mathrm{k_2 \times n_2}}}_{\text{Platform GF matrix}} \tag{4}$$

where $\mathbf{S}_{\mathrm{mn}}^{\mathrm{k_m \times k_n}} = \mathbf{B}_{\mathrm{m3}}^{\mathrm{k_m \times n_3}} \cdot \left[\mathbf{A}_3^{\mathrm{BE}}\right]^{-1} \cdot \mathbf{B}_{\mathrm{3n}}^{\mathrm{n_3 \times k_n}}$. The $\mathrm{n_1}$, $\mathrm{n_2}$, $\mathrm{n_3}$ are the number of BE unknowns on antennas and platform. The $\mathrm{k_1}$ and $\mathrm{k_2}$ are skeleton BE unknowns. The complexity for assembling the PGF matrix is $O((\mathrm{k_1} + \mathrm{k_2}) \cdot \mathrm{n_3})$.

During the design stage (online computing), Eq. 2 is solved by Krylov iterative methods with an additive Schwarz preconditioner [3]. We remark that the online computing complexity does not depend on the size of the in-situ platform. As a result, in-situ design and optimization of multi-antenna systems can be performed at the same cost as the free-space radiation.

III. NUMERICAL RESULTS

(i) Offline calculation: We consider four antennas mounted on the ship's mast as shown in Fig. 2. In the offline computation, we first generate separate BE meshes on the exterior surface of antennas and the ship platform. The PGF matrices are then constructed and assembled in the AR-ID representation as in Eqs. 3 and 4. We note that the PGF matrices are introduced to characterize the coupling with the platform only, and separately compressed with the free-space GF matrices. Thereby, the rank of PGF matrices are extremely low, as depicted in Fig. 3. Moreover, the free-space GF matrices can still be compressed with the fast multipole method.

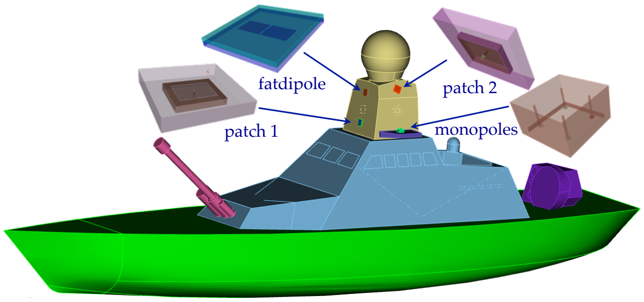
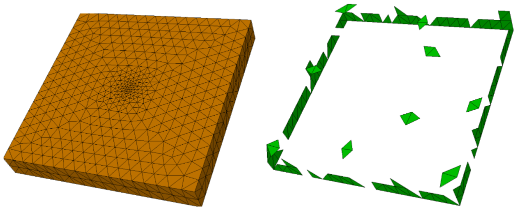


Fig. 2: Example of antennas mounted on ship's mast.



(a) $\mathrm{n_1}$=1669 (b) $\mathrm{k_1}$=147

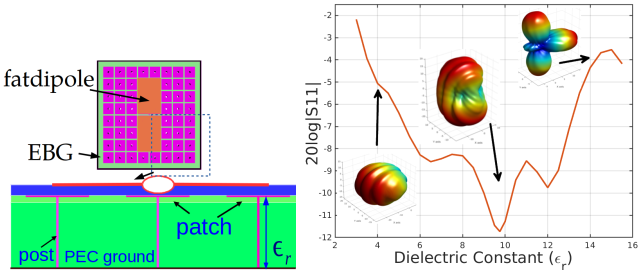
Fig. 3: Data-sparse representation of fatdipole's PGF.

(ii) Online computing: In the online phase, we generate the volume meshes for individual antennas independently. The FE matrices are then combined with the GF matrices obtained in the offline phase. The simulation results are presented in Table I, comparing to the reference DD method [3]. The proposed work exhibits near 300 times speed-up in runtime, since its online computing complexity is platform independent.

TABLE I: Simulation results (reference / this work)

Excitation	fatdipole	patch1	patch2	monopole
S-parameter	0.81 / 0.81	0.44 / 0.42	0.45 / 0.45	0.56 / 0.56
DD iterations	80 / 2	59 / 2	52 / 2	41 / 1
Runtime (s)	1712 / 4	1297 / 5	1061 / 4	852 / 3

(iii) Many-query design: Attributed to the rapid time-to-solution, we can perform the platform-aware in-situ antenna design. As an application, we replace the simple fatdipole antenna by a metasurface fatdipole antenna [4], then sweep the dielectric constant (ϵ_r) of substrate to find the optimal performance. All can be done with the same PGF at the exterior surface. It takes 6s on average for individual simulations and the results are illustrated by Fig. 4.



(a) metasurface antenna (b) Dielectric constant sweeping

Fig. 4: In-situ design of metasurface fatdipole antenna.

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