Higher order, Globally Constraint-preserving DGTD schemes for CED

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The time-dependent equations of computational electrodynamics (CED) are evolved consistent with the divergence constraints on the electric displacement and magnetic induction vector fields. Respecting these constraints globally has proved to be very useful in the classic finite-difference time-domain (FDTD) schemes. Prior generations of discontinuous Galerkin schemes for CED respected these constraints within an element, but not globally. As a result, there has been a recent effort to design discontinuous Galerkin time domain (DGTD) schemes that satisfy the same global constraints and, nevertheless, draw on recent advances in higher order Godunov methods. Here we catalogue the design of globally constraint-preserving DGTD schemes. The algorithms presented here are based on a novel DG-like method that is applied to a Yee-type staggering of the electromagnetic field variables in the faces of the mesh. The other two novel building blocks of the method include constraint-preserving reconstruction of the electromagnetic fields and multidimensional Riemann solvers; both of which have been developed in recent years by the first author.

The resulting DGTD scheme is linear, at least when limiters are not applied to the DG scheme. As a result, it is possible to carry out a von Neumann stability analysis of the entire suite of DGTD schemes for CED at orders of accuracy ranging from second to fourth. The analysis requires some simplifications in order to make it analytically tractable, however, it proves to be extremely instructive. A von Neumann stability analysis is a necessary precursor to the design of a full DGTD scheme for CED. It gives us the maximal CFL numbers that can be sustained by the DGTD schemes presented here at all orders. It also enables us to understand the wave propagation characteristics of the schemes in various directions on a Cartesian mesh. We find that constraint-preserving DGTD schemes permit CFL numbers that are competitive with conventional DG schemes. However, like conventional DG schemes, the CFL of DGTD schemes decreases with increasing order. To counteract that, we also present constraint-preserving PNPM schemes for CED. We find that the third and fourth order constraint-preserving DGTD and P1PM schemes have some extremely attractive properties when it comes to low-dispersion, low-dissipation propagation of electromagnetic waves in multidimensions. Numerical accuracy tests are also provided to support the von Neumann stability analysis. We expect these methods to play a role in those problems of engineering CED where exceptional precision must be achieved at any cost.

It is well-known that the limiting step in DG schemes causes a reduction of the optimal accuracy of the scheme. In this paper we document simulations where permittivity and permeability vary by almost an order of magnitude without requiring any limiting of the DG scheme. This very favorable finding ensures that DGTD schemes retain optimal accuracy even in the presence of large spatial variations in permittivity and permeability. We also study the conservation of electromagnetic energy in these problems. Our further finding is that the electromagnetic energy is conserved very well even when permittivity and permeability vary strongly in space; as long as the conductivity is zero.