

Higher order, Globally Constraint-preserving FVTD schemes for CED

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The Finite Difference Time Domain (FDTD) scheme has served the computational electrodynamics community very well and part of its success stems from its ability to globally satisfy the constraints in Maxwell's equations. Even so, we present a higher order accurate Finite Volume Time Domain (FVTD) Godunov scheme for computational electrodynamics (CED) which satisfies all the same constraints and simultaneously retains all the traditional advantages of Godunov schemes. In this paper we present FVTD schemes for CED in material media with high order of accuracy.

From the FDTD method, we retain a somewhat modified staggering strategy of primal variables which enables a very beneficial constraint-preservation for the electric displacement and magnetic induction vector fields. This is accomplished with constraint-preserving reconstruction methods which are extended in this paper to third and fourth orders of accuracy. The idea of one-dimensional upwinding from Godunov schemes has to be significantly modified to use the multidimensionally upwinded Riemann solvers developed by the first author. In this paper, we show how they can be used within the context of a higher order scheme for CED.

We also formulate very efficient ADER timestepping strategies to endow our method with sub-cell resolving capabilities. As a result, our method can be stiffly-stable and resolve significant sub-cell variation in the material properties within a zone. Moreover, we present ADER schemes that are applicable to all hyperbolic PDEs with stiff source terms and at all orders of accuracy.

Second, third and fourth order accurate schemes for numerically solving Maxwell's equations in material media are presented in this paper. Several stringent tests are also presented to show that the method works and meets its design goals even when material permittivity and permeability vary by an order of magnitude over just a few zones. Furthermore, since the method is unconditionally stable and sub-cell-resolving in the presence of stiff source terms (i.e. for problems involving giant variations in conductivity over just a few zones), it can accurately handle such problems without any reduction in timestep. We also show that increasing the order of accuracy offers distinct advantages for resolving sub-cell variations in material properties. Most importantly, we show that when the accuracy requirements are stringent the higher order schemes offer the shortest time to solution. This makes a compelling case for the use of higher order, sub-cell resolving schemes in CED.