Accelerated Cartesian Harmonics based framework for acceleration of retarded potentials

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Methods for solving time domain integral equations have grown by leaps and bounds over the the past two decades. Specifically, significant advances have been made in ameliorating memory and computational complexity, thanks to development of both the plane wave time domain (PWTD) (A. A. Ergin, B. Shanker, and E Michielssen, Journal of Computational Physics 146 (1), 157-180, 1998) and adaptive integral equation methods (AIM) (A. E. Yilmaz, J. M. Jin, and E. Michielssen, IEEE Transactions on Antennas and Propagation, 52, 2692-2708, 2004). These methods reduce the cost to $\mathcal{O}(N_t N_s \log^2 N_s)$, and $\mathcal{O}(N_t N_c \log^2 N_c)$, where N_s is the number of spatial unknowns, N_t is the number of temporal unknowns, and N_c is the number of auxiliary in grid points on the Cartesian grid. The second main bottleneck was late time stability; again, research in overcoming this hurdle has been extensive (A. J. Pray *et. al.*, IEEE Transactions on Antennas and Propagation, 62, 6183-6191, 2014), (Chen *et. al.*, Comm. Comput. Phys. 11, 383-399, 2012), it is now well understood and there exist methods that provide reliable results. While this summarizes the state of the art, there is still an unsolved problem. The integration of these "fast" methods with "stable" methods has not been achieved. This is largely due to different methods used in the two regimes. Specifically, stable time domain methods use different approximations to effect interaction between basis functions. In Pray *at. al.*, the convolution of the space-time basis function with the retarded potential is approximated using a set of polynomials that is smooth and continuous over the domain of the triangle. Whereas in PWTD, currents are mapped onto approximate prolate spheroidal wave functions, and then the retardation is effected via a plane wave or a spectral expansion. However, key challenge in using a spectral expansion arises from evaluation of the spectral integral. To accurately evaluate the integral numerically, one needs sample at a rate proportional to the maximum frequency content of the sampled signal. This implies that given a time step size and the corresponding sampling rate f_s, the maximum frequency content is f_s. This rate, in turn, controls the number of terms necessary to evaluate the spectral integral. Failure to evaluate the spectral integral accurately leads to errors that result in late time instability. Unfortunately, sampling at this high rate renders the method untenable in that while it scales as before, the constant is very high.

This research attempts to find an alternative. Our approach relies on spatial sampling. As a preliminary step, we start with Accelerated Cartesian Expansions (B. Shanker and H. Huang, Journal of Computational Physics, 226 (1), 732-753, 2007) to derive a method that can be used to accelerate retarded potentials. Unlike our previous attempt which was intimately tied to time step separation between box pairs and intended to overcome low frequency bottleneck, our intention is to develop a method that relies more on the method used for stabilization. Using this, we derive recursion relations for translation operators and demonstrate the accuracy of the proposed approach. Further, we hope to implement a 1-level scheme within an time domain integral equation framework and investigate stability.