Dispersion Engineered Space-Time Modulated Transmission Line

Donald DiMarzio (donald.dimarzio@ngc.com)[1], Stéphane Larouche (stephane.larouche@ngc.com)[1], Philip Hon (philip.hon@ngc.com)[1] and Vesna Radisic (vesna.radisic@ngc.com)[1]

[1]NG Next, Northrop Grumman Corporation

One Space Park Redondo Beach, CA 90278 USA

Breaking Lorentz reciprocity is key to many current and future technological applications [1]. Nonreciprocal isolators and circulators have relied on bulk magnetic materials to break spatial symmetry to control the flow of RF energy [2]. Nonmagnetic alternatives for reciprocity breaking are needed for compact isolator integration. The last few years have seen a dramatic growth in the development of concepts for wideband and low loss nonmagnetic isolators that hold the possibility for RF systems integration.

Periodic space-time modulation of nonlinear transmission lines has been identified as a method for breaking reciprocity. Originally motivated by parametric amplification, the foundations of periodic space-time modulated media were established in work by Slater [3], Tien and Suhl [4]. Later, Cassedy, Oliner and others performed more rigorous studies using transmission line theory [5 6 7]. A more rigorous solution based on Hessel and Oliner's work for nonreciprocity was done by Taravati, Chamanara and Elnaggar using Bloch-Floquet theory [8 9 10 11]. This work was done only for *nondispersive continuous* right-handed media. It was recognized that dispersion engineering can be used to control and possibly enhance isolator characteristics. In this paper we explored the effects of the natural dispersion arising from a realistic finite unit cell size on the non-reciprocal properties of a right handed space-time modulated transmission line.

A right handed one-dimensional transmission line structure with independently modulated capacitance (e.g., varactors) C_n, and a constant inductance L per unit cell, is shown in figure 1. The Floquet theorem states that a general solution to the wave equation in an infinite medium that is modulated via a periodic potential can be written as a product of a plane wave and a function with the same periodicity of the modulation.

Using Kirchoff's law we derive the wave equation in terms of voltage V(n, t) as a function of cell position n and time t,

$$L\frac{d^2}{dt^2}[C(n,t)V(n,t)] = V(n+1,t) + V(n-1,t) - 2V(n,t) \tag{1}$$

As was done in previous studies[6 7 11], we take the periodic modulation of C to be given by the simple expression

$$C(n,t) = C_0 + C_m\cos[w_m t - k_m na] \tag{2}$$

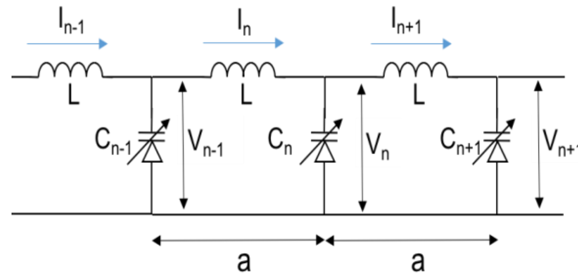

Figure 1 Right-handed transmission line circuit diagram with independently modulated capacitances (varactors) and

where C_0 is a constant background capacitance, C_m is the capacitance modulation, and na equals the position z of the unit cell. Using the Floquet solution, we derive the following recursion relation among the Floquet expansion coefficients V_s,

$$V_{s-1} + D_s V_s + V_{s+1} = 0 \tag{3}$$

with

$$D_s = 2/M\,[1 - 4/LC_0\,[\sin^2[(k + sk_m)a/2]/(w + sw_m)^2] \tag{4}$$

where the modulation factor $M = C_m/C_0$.

D_s is the generalized recursion term for a right-handed transmission line with a finite unit cell length of a. This results in a natural dispersion that was not present in the previously studied continuous (a = 0) transmission lines, and furthermore imposes an expected upper cut-off frequency. For small values of a, equation 6 reduces to the previously derived expressions for a space-time modulated continuous right-handed transmission line[6 7].

As a example of what effects the finite unit cell size has on the non-reciprocal properties, Figure 2 shows dispersion w-k grid plot solution (bright lines) to Equations 3 and 4 for this transmission line with a particular set of modulation parameters. Here $k_N = 2\pi k/k_m$, $w_N = 2\pi w/k_m v_0$, $p = k_m a/2\pi$, $r = v_m/v_0$, $v_m = w_m/k_m$ and $v_0 = a/\sqrt{LC_0}$. v_0 represents the phase velocity of the signal wave for no space-time modulation and with the signal wavelength large compared to the unit cell length a, v_m is the equivalent phase velocity of the space-time modulation wave, and the r factor is the ratio of these two velocities.

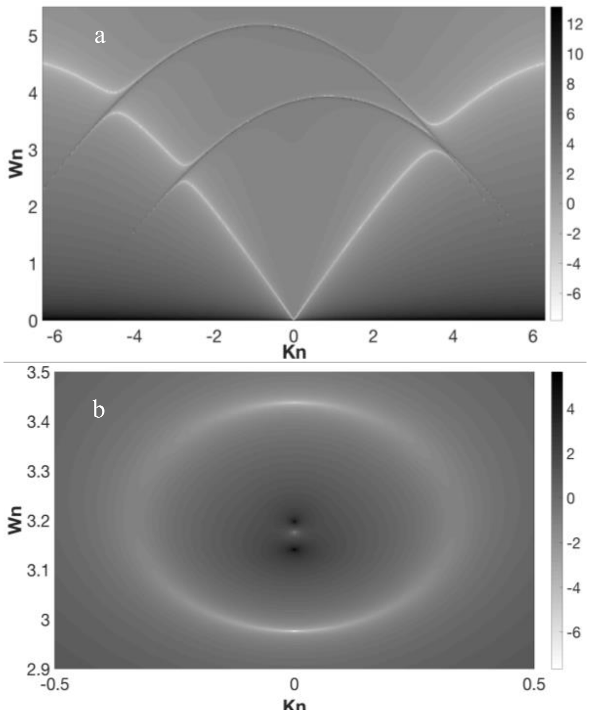

Figure 2 a) Space-time modulated transmission line dispersion with finite unit cell, and b) Imaginary k_N for +k propagation and fixed Real k_N = 3.553. M = 0.20, r = 0.10 and p =0.44. p factor chosen to merge double gap for +k signal propagation.

The newly introduced parameter p is equivalent to the ratio of the transmission line unit cell length a to the spatial modulation wavelength. The p factor introduces an additional control over the overall system dispersion and results in both multi gap dispersion and enhancements of harmonic contributions to the total wave solution. In general, the introduction of a finite unit cell size results in a non-reciprocal double gap structure in the dispersion diagram for both forward and backward propagation signal waves. The gap separation is tunable via adjustment of the r and p parameters. In figure 2 the parameters are adjusted so that the double gap

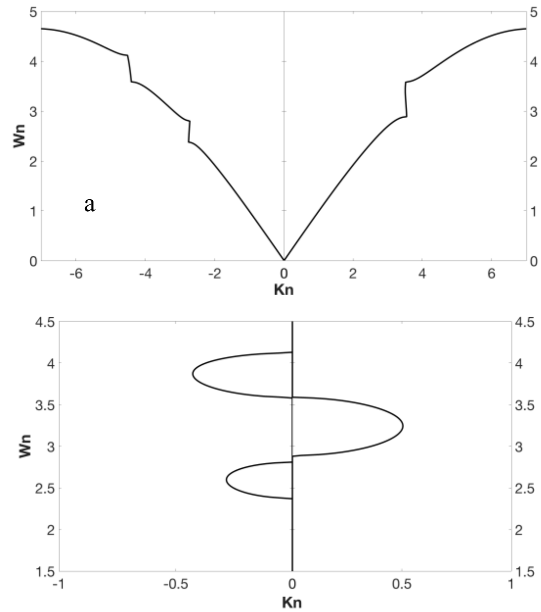

Figure 3 Calculated band structure for non-reciprocal finite unit cell transmission line for M = 0.30, r = 0.10 and p =0.44, a) dispersion curves and b) Imaginary k_N values with the bandgaps for fixed Real k_N's.

merges for forward propagating signals, with a double gap for backward signals. Figure 3 illustrates this more clearly showing the explicit dispersion solution.

For this particular choice of transmission line parameters, we are able construct what is essentially a bandpass filter with non-reciprocal properties. In this example, a signal wave can propagate in the **-** k direction between the two gaps depicted on the left side of Figure 3b, but signals traveling in the + k direction within this band region are effectively blocked. This behavior can be automatically reversed by reversing the direction of the modulation pump wave.

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