Gabor Frame-Based Sparsification and Radiation Boundary Conditions for Parabolic Wave Equations

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Parabolic equations (PEs) have been used extensively to model electromagnetic wave propagation over complex terrain, including rural, ocean, and urban environments (Levy, Parabolic Equation Methods for Electromagnetic Wave Propagation, The Institution of Engineering and Technology, 2009). There remains, however, an active need for improving the computational efficiency of PE solution techniques, particularly in applications such as simulation of non-line-of-sight communications. Present PE solution techniques either leverage finite-difference or Fourier transform propagators. Among all available schemes, the so-called split-step Fourier transform method has been widely adopted because of its ease of implementation and ability to handle wide-angle propagation (Thomson et al, "A wide-angle split-step algorithm for the parabolic equation," *J. Acoust. Soc. Am.*, 1983).

Split-step Fourier methods, however, remain problematic when applied to large, open domains. Since the split-step scheme requires successive Fourier transforms at each spatial step, the memory and CPU time per step scale as $O(N)$ and $O(N \log N)$ where N is the vertical domain size. The method does not naturally take advantage of spatial or spectral sparsity of the fields; in other words, it wastes resources processing structured fields. Additionally, while periodic and Dirichlet/Neumann boundary conditions can be very easily implemented, the same cannot be said for radiation boundary conditions (RBCs). Perfectly-matched layers have been used to achieve RBCs in finite-difference propagators (Berenger, "A perfectly matched layer for the absorption of electromagnetic waves," *J. Comput. Phys.*, 1994.), but for split-step Fourier transform methods, RBCs are typically emulated with more classical methods such as absorbing layers (B. Engquist and A. Majda, "Absorbing boundary conditions for the numerical simulation of waves," *Math. Comput.*, 1977). Absorbing layers, however, become expensive at high propagation angles, and other RBC implementations are often only suited for special cases such as short-range scattering or line-of-sight propagation.

This paper proposes to use Gabor frames to lower the memory and CPU costs of split-step Fourier PE solvers and allow for the easy implementation of RBCs. Gabor transforms have been used before for PE-based modeling of geophysical problems (Chen et al, "Target-oriented beamlet migration based on Gabor-Daubechies frame decomposition," GEOPHYSICS, 2006), but they have not yet been extensively used in electromagnetics. The Gabor transform decomposes a wavefront into an overcomplete set of smooth, locally supported frame functions, consisting of spatially shifted and frequency modulated windows and easily exploits sparsity in the space-frequency representation of structured fields (e.g. beams or fields due to point scatterers). By precomputing the propagation characteristics of each frame function, a Gabor transformed wavefront can be efficiently propagated from one PE slice to the next. Indeed, the choice of using smooth window functions ensures beam-like propagation and minimal spreading for each frame function, leading to a sparse propagation matrix. The CPU time per step therefore is limited to the number of nonzero elements of the sparse propagation matrix addressed by the sparse space-frequency representation of the fields. RBCs are trivially implemented by removing frame functions that propagate outside the computational domain (i.e. beyond a certain height bounds) from consideration, a feat that is impossible using classical split-step Fourier methods.

Gabor-frame enhanced PE solvers have been successfully implemented in 2D, and 3D implementations are pending. Numerical results demonstrating the memory and CPU efficiency of the new solvers will be provided at the conference.