A Refinement-Free Calderon preconditioner for the Symmetric Formulation of the EEG Forward Problem

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Electroencephalography (EEG) is a widespread neuroimaging tool that provides high temporal resolution readings of the brain activity starting from non-invasive measurements of electric potential at the scalp. In its high resolution incarnations, using inverse source algorithms, it is possible to reconstruct the neuronal current distribution responsible for the potential reading of the electroencephalograph. This, however, requires multiple solutions of the EEG forward problem, where the potential at the scalp is obtained given a known current distribution in the head volume. Given its importance for high imaging quality, several efforts have been profused by both computational science and bio-medical scientific communities for developing schemes that improve and stabilize the EEG forward problem. Among the different formulations to solve the EEG forward problem, the symmetric formulation (Kybic, J., et al., A common formalism for the integral formulations of the forward EEG problem, IEEE transactions on medical imaging, 24(1), 12-28.) is widely used. This formulation is integral equation-based, which has shown robustness and high accuracy in comparison to other existing approaches. Unfortunately, this method is of first kind in nature. This fact results in severe numerical instabilities and ill-conditioning every time realistic models, such as those obtained from MRI, are used.

Recently, a new Calderon-like strategy was originated to precondition the EEG symmetric formulation (Ortiz G., J. E., et al., A Calderon regularized symmetric formulation for the electroencephalography forward problem, Journal of Computational Physics, 375, 291-306.). This preconditioner, based on block Calderon identities, delivered an equivalent but stable second kind formulation, thus allowing for a fast convergence of the solution independently of the mesh parameters. As other Calderon approaches, however, this formulation required the use of a barycentrically refined mesh and this requirement resulted in some additional computational overhead. In another very recent work, a new strategy for Calderon preconditioning was presented for the Electric Field Integral Equation, not requiring the barycentric refinement of the mesh (Adrian, S. B., et al., On a refinement-free Calderon multiplicative preconditioner for the electric field integral equation. Journal of Computational Physics, 376, 1232-1252.).

In this work we extend the latter approach to the case of EEG block formulation matrices and we present a new symmetric formulation which not only will be numerically stable and well conditioned, but differently from standard Calderon strategies, it will not require the use of the barycentrically refined mesh. This will be obtained by leveraging on the equivalence spectral properties of the Laplace-Beltrami operator and constructing suitable block operators of inverse orders with respect to the symmetric formulation.

Theoretical developments will be presented, together with several numerical studies showing the practical effectiveness of all newly obtained techniques.