A Krylov-Subspace-Exponential Time Integration Scheme for 3-D Discontinuous Galerkin Time-Domain Methods

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Discontinuous Galerkin time-domain (DGTD) methods have gained much attention in the computational electromagnetics community due to their capability of combining unstructured meshes, hp-adaptivity and efficient explicit time integration. However, in simulations involving electrically small structures, the permitted time-step size for explicit time integration schemes has to compromise to the finest mesh and is thus extremely small. Implicit time integration schemes, on the other hand, can relax the limit of the time-step size but require solutions of large linear systems in every time advance and the accuracy is not satisfying under large time-step size. The exponential time integration (ETI) methods have been proposed as potential alternatives of implicit time integration schemes due to their unconditional stability and high accuracy under large time-step size. Unluckily, the explicit evaluation of the matrix exponentials often spoils the sparsity of the coefficient matrices, leading to prohibitive memory costs.

In this paper, we propose for three-dimensional DGTD methods a Krylov-subspace-based ETI (KETI) scheme, which approximates the new-time solution of time-dependent Maxwell's equations by its projection on Krylov subspace iteratively. The proposed scheme can be implemented in a complete matrix-free manner, i.e. the dense matrix exponentials are absorbed in matrix-vector products constituting Krylov subspace and don't require explicit evaluation, hence the huge memory costs can be avoided. In addition, the shift-and-invert (SAI) technique is employed to cluster the spectrum of the coefficient matrices to improve the convergence rate of Krylov iterations. Since in every iteration of the resultant SAI-KETI scheme the action of the inverse of a known matrix on a given vector is demanded, a restricted additive Schwarz (RAS) preconditioner is further incorporated to accelerate this process. The RAS preconditioner is a domain-decomposition method in nature and can fully exploit the block structure of the coefficient matrices arising from DG methods. Last but not least, existing ETI schemes either require multiple matrix exponential evaluations per time step or have only second order of accuracy in the presence of impressed sources. In this paper, we focus on linear time-invariant media and present a novel treatment of the inhomogeneous source terms, which provides any desired order of accuracy at ignorable extra costs in the pre-processing procedure. Numerical experiments demonstrate the accuracy and efficiency of the proposed RAS-SAI-KETI time integration scheme over those of explicit and implicit time integration schemes.