Fast Surrogate Model-Assisted Uncertainty Quantification via Quantized Tensor Train Decompositions

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Advances in computational methods and computing capabilities have enabled the fast quantification of uncertainties of systems composed of electromagnetic (EM) and circuit components. Methods for fast uncertainty quantification (UQ) characterize variability of the systems' observables due to the uncertainties in the systems' excitation or physical configuration. UQ methods for example can be used to compute the variability in the input impedance of a circuit due to manufacturing tolerances of the lumped elements, or the variability in the center frequency of an antenna due to temperature variations during its operation. Traditional UQ methods include adjoint sensitivity methods, analytical probabilistic models, and Monte Carlo (MC) methods. Unfortunately, these methods either only apply to stochastic problems with limited variations in the systems' input parameters or require a large number of simulations. Alternative surrogate model-assisted methods leveraging generalized polynomial chaos (gPC) expansions and sparse grid (SG) interpolation schemes are receiving significant attention due to their wider applicability, efficiency, and accuracy. These methods construct surrogate models for the observables (e.g., fields, voltages, and currents) by judiciously selecting a small number of points in the random domain over which the random variables parameterizing the uncertain input parameters are defined. Then, they compute the statistics of observables via MC using the surrogate models in lieu of the full-wave EM simulators. Unfortunately, when the number of random variables is large, gPC and SG generatedsurrogate models lack accuracy. To tackle this problem, high dimensional model representation (HDMR) techniques and anisotropic sparse grid (ASG) techniques have been proposed. These techniques are effective when the observables can be represented in terms of the random variables' individual contributions and low-order combined contributions. Nevertheless, they lose their efficiency/accuracy for observables that are high-order and/or combined functions of the random variables. The latter is often the case when analyzing stochastic EM problems.

In this study, the quantized tensor train (OTT) decompositions (Osedelets, Const. Approx., 37(1), 1–18, 2013) generated by density renormalization group cross approximation (DMRG-Cross) (Savostvanov and Osedelets, NDS 2011., 7, 1–8, 2011) are proposed for efficient UQ of coupled EM and circuit systems. The proposed method generates compressed representations (i.e., QTT decompositions) of the highdimensional tensor grid of observable values required to generate gPC expansions. The QTT decomposition is generated by adaptively executing a full-wave simulator for a select set of EM and circuit system realizations, and generating the rest of the high-dimensional tensor grid of observable values through adaptively constructed interpolation rules. Like the functional tensor train decompositions (Bigoni et al., arXiv, 1405.5713, 2014), the proposed QTT decomposition is more accurate than those produced via HDMR and ASG techniques. Furthermore, the number of full-wave simulations required to obtain an accurate QTT representation increases linearly with the number of random parameters and logarithmically with the polynomial order of the gPC expansion. In other words, unlike ASG and HDMR methods. OTT does not suffer from the curse of dimensionality, and incurs minimal computational cost while increasing the order of the approximation. In the presentation, we will demonstrate the efficiency and accuracy of the QTT method by comparing its performance with those of HDMR and ASG techniques for a number of illustrative stochastic EM examples.