Physics-Oriented Statistical Analysis of Information Transmission in Wave-Chaotic Environments

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***Abstract*—The goal of this work is to study the channel capacity and coherence of communication in wave-chaotic environments, through the investigation of a physics-oriented statistical wave model. The objective is attained by cutting across traditional disciplinary boundaries between electromagnetic theory, wave chaos physics, random statistical analysis and information theory. The methodology is to first establish fundamental statistical representations of complex wave-chaotic media, then integrate component-specific features of transmitters and receivers, and finally encode the governing physics into the mathematical information theory. The theoretical research is evaluated and validated through representative experiments.**

I. INTRODUCTION

Electromagnetic (EM) field theory provides the fundamental physics of wireless communications. Over the past decades, EM theory has played a significant role in the design, performance assessment, and deployment planning of wireless devices and systems. Meanwhile, wireless communications are taking place in increasingly congested, contested, and competitive environments. Particularly, communication in wave-chaotic environments is a topic of both fundamental and practical importance [1]–[3]. Applications include indoor radio channels, dense urban cells, transmission through diffusive random media, and disordered media, etc.

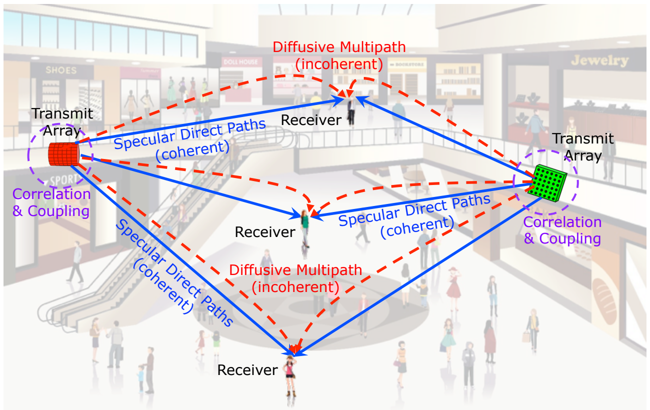


Fig. 1: Multiuser MIMO in rich scattering environments.

A generic problem statement is shown in Figure 1, where the multiuser multiple-input multiple-output (MIMO) communication in a confined environment is illustrated. The complexity of the propagation environment and antennas poses grand challenges in existing semi-empirical statistical channel models. Among many other issues, we mention effects of mutual coupling in MIMO antennas [4], correlated random field in small-scale fading [5], mixed specular direct path and diffuse multipath, and site-specific, short-orbit coupling.

II. METHODOLOGY

This paper presents an original contribution for analyzing the physical layer of communication in wave-chaotic environments. The proposed statistical model rigorously resolves the spatial correlation, propagating coherence, and antenna mutual coupling, from first-principles calculations. The outcomes achieve an imperative theory-driven, design-under-chaos capability for optimizing information transmission. The methodology is described as follows:

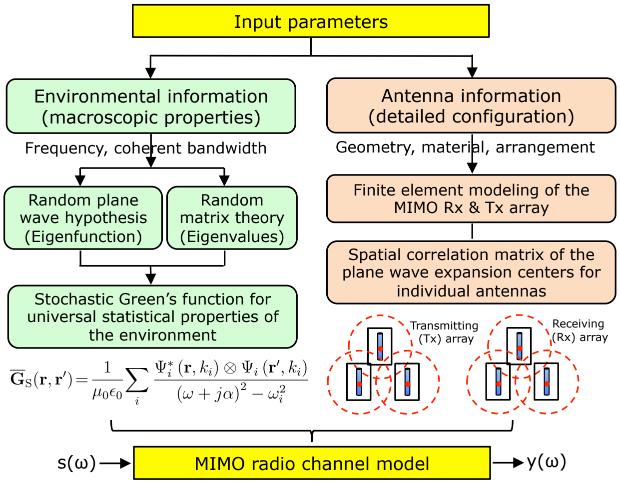


Fig. 2: Illustration of the modeling methodology.

<u>1) Environmental aspects</u>: Modeling the underlying physics of propagation is essential to the study of information transmission. It has been discovered that, the ergodic modes in chaotic environments lead to certain universal, structureless statistical behavior of EM fields. It motivates the investigation of the stochastic Green's Function (SGF) method, which is an innovative theoretical solution to Maxwell's Equations in complex wave-chaotic media. The SGF is constructed from the eigenfunction expansion with eigenvector statistics derived from Berry's random wave model [6] and eigenvalues statistics generated by Wigner's random matrix theory [7]. The generic statistical properties of the SGF is characterized by a few macroscopic parameters, including operating frequency, coherence bandwidth, and mean eigen-spacing.

<u>2) Antenna aspects</u>: The realistic transmission performance is also determined by wireless devices and antennas. With the

emergence of the ultra-dense networks and massive MIMO, there is a pressing need to accuratly characterize the spatial correlation and interference in wireless channels. To analyze the mutual coupling among antennas, we use the finite element method to model the volumetric domain of antennas. The surface of the antennas is discretized with the SGF-kernel integral equation (IE) method. The plane wave spectrum correlation among antenna elements is determined by the translation operator between the eigen-expansion centers.

The outcome provides a statistical ensemble of S-parameter matrices describing the transmission performance. We can thereby obtain the random transfer (channel) matrix, evaluate the channel capacity, optimize the coding and beamforming.

III. EXPERIMENTAL VALIDATION AND VERIFICATION

(**i**) Uncorrelated MIMO Channel: We begin with analyzing the ergodic capacity of MIMO channel for different numbers of well-separated transmitting (Tx) and receiving (Rx) antennas. Since individual antennas are far apart, the SGF will be dominated by its incoherent, diffuse multipath component. The resulting channel matrix elements are completely random and uncorrelated. The computed cumulative distribution function (CDF) of capacity is plotted in Fig. 3, comparing to the analytical, i.i.d. Rayleigh fading model. The results verify the work successfully predicts the performance of uncorrelated MIMO channels in the multipath chaotic environment.

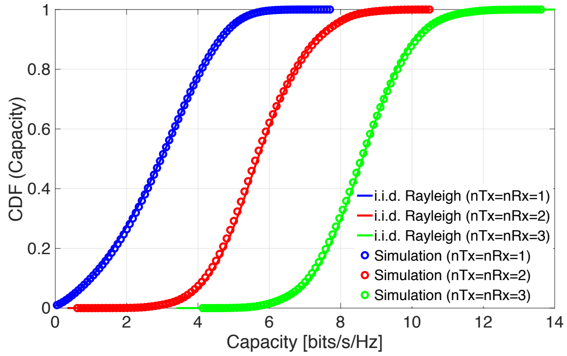
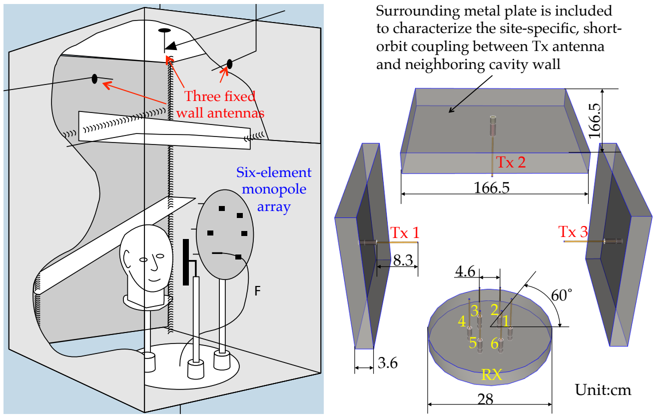


Fig. 3: CDF of MIMO capacity at SNR = 10dB.

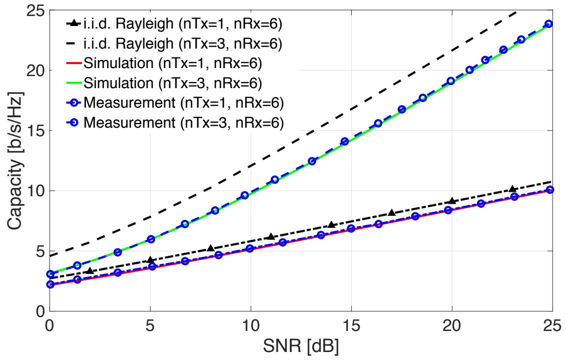
(**ii**) Correlated MIMO Channel: We proceed to study the case of correlated MIMO channel. Shown in Fig. 4(a) is the reverberation chamber (RC) experiment presented in [8] for measuring a six-element monopole circular MIMO array. The monopole array is located on a circular metal plate with radius $0.14m$. The spacing between monopole antennas is 0.24λ at 900MHz. Our computational model in Fig. 4(b) only needs three wall antennas (Tx) and six-element monopole array on the metal plate (Rx). The spatial multipath propagation between them is characterized by the SGF - IE method. The computed channel capacity is compared to the measurement in Fig. 4(c), where a very good agreement is observed.

IV. CONCLUSION

We present a novel mathematical/statistical model analyzing the information transmission in multipath, chaotic environments. It rigorously characterizes the antenna mutual coupling,



(a) RC configuration [8] (b) Computational setup



(c) Mean channel capacity

Fig. 4: Information capacity of correlated MIMO channel.

and resolves the transmitting, propagation, and receiving correlations in the wave propagation. Such model does not appear to be available in the literature. The work has diverse applications to wireless experimentation, time-reversal experiments, wavefront shaping, and sensing and targeting.

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