# An Equivalent ABCD-Matrix Approach for Multilayer Wire-Medium-Type Structures


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It is well known that at microwave frequencies (even in the very long wavelength limit), wire media is characterized by strong spatial dispersion, resulting in nonlocal constitutive relations between the macroscopic fields and the electric dipole moment. To take into account spatial dispersion in the electromagnetic interaction with wire-medium (WM) structures, a nonlocal homogenization formalism has been developed (see, for example, M. G. Silveirinha, C. A. Fernandes, and J. R. Costa, IEEE Tran. Antennas Propag., 56, 405-415, Feb. 2008), which necessitates the use of additional boundary conditions (ABCs) at the WM interfaces (see, for example, S. I. Maslovski, T. A. Morgado, M. G. Silveirinha, C. S. R. Kaipa, and A. B. Yakovlev, New J. Phys., 12, 113047, 2010). Various WM-type topologies have been studied with application to negative refraction, subwavelength imaging, partial focusing, absorption, among others. However, specifically concerning multilayer WM-type structures with (in general) arbitrary impedance surfaces at the WM interfaces, there is no analytical model available in the literature.

Here, we propose a simple analytical model based on the ABCD-matrix approach for an equivalent WM interface. Two semi-infinite structures are considered with the equivalent WM interfaces: (i) semi-infinite local dielectric-nonlocal WM and (ii) two semi-infinite nonlocal WM. The ABCD (transmission) matrices for equivalent interfaces are retrieved by imposing the conventional and additional (generalized) boundary conditions at the WM interfaces depending on the WM termination. The ABCD matrix of an equivalent interface captures the nonlocal effects due to the extraordinary transverse magnetic (TM) WM mode, and in part the nonlocal effects due to the extraordinary transverse electric magnetic (TEM) WM mode, with the rest of the WM nonlocal material supporting only the TEM WM mode. This enables one to characterize the overall response (reflection/transmission for obliquely incident plane waves or near field due to canonical sources) by cascading the ABCD matrices of equivalent WM interfaces and WM slabs as transmission lines supporting the only propagating TEM WM mode, resulting in simple circuit-model formalism for bounded WM structures with arbitrary terminations, including the open-end, patch/slot arrays, thin metal/2D material, among others. It should be noted that the ABCD matrices for equivalent WM interfaces (which only depend on the WM parameters) in general neither ensure the conservation of energy nor reciprocity, and therefore, the interfaces behave as non-reciprocal lossy or active systems. However, the overall response of a multilayer WM structure is consistent with the lossless property (maintaining conservation of energy and reciprocity). This is explained by the fact that in the non-local WM the Poyinting vector has an additional correction term which takes into account the non-local effects. Results are obtained for various numerical examples demonstrating a rapid and efficient solution for geometrically complex multilayer WM structures with arbitrary terminations, subject to the condition that the WM interfaces are decoupled by the evanescent TM WM mode below the plasma frequency.