Transmission Line Models of Planar Slot Antennas

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***Abstract*—We propose a systematic approach to describe planar slot antennas, embedded in generic stratified media. An equivalent transmission line model for the slot is proposed, based on a spectral domain analysis. First, we introduce a method of moments solution to model semi-infinite slots, fed by a delta-gap excitation. The solution entails only two basis functions, one located at the feed and the other at the termination. The latter basis function is chosen to properly account for the field diffractive behavior at the antenna end point. An approximate circuit model is then introduced, which describes the main mode propagating along the slot as an equivalent transmission line. Lumped impedances are extracted to accurately describe the source and the end point. This procedure can be used to derive the input impedance of planar antennas with arbitrary length in generic layered media or the interaction between multiple feeds within the same slot.**

***Index Terms*—Equivalent circuit, input impedance, slot antenna.**

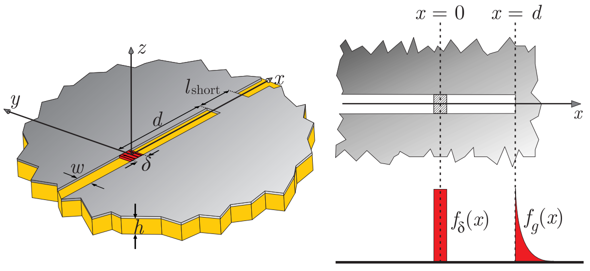
I. INTRODUCTION

A convenient way to describe a center-fed slot is by an equivalent transmission line model, where the excitation is modeled as a shunt generator and the slot arms are represented as two transmission line sections. Transmission line models for slot antennas were given in [1], [2], which considered short circuits to describe the slot terminations, thus did not account for the reactance associated with the end points. An improved model was proposed in [3], where the inductance of the slot shorted ends was considered. However, all the existing models do not account for the reactance of the feed and the diffraction from the edge. Moreover, the radiation is modeled as a distributed resistance through a lossy line or as a single lumped resistance.

A different approach is presented here, where an improved model is proposed that accurately describes the reactive nature of both the feed and the terminations of the slot. This is a Method of Moments (MoM) solution that includes only two basis functions, one associated with the feed point and one located at the termination. This solution can be conveniently approximated with an equivalent transmission line with good accuracy and improved physical insight.

II. MOM SOLUTION FOR SEMI-INFINITE SLOT

Let us consider a narrow, x-oriented, infinite slot fed by a delta-gap excitation, which is small in terms of the wavelength. The slot is shorted at distance d from the feed, as shown in Fig. 1(a). The short is approximated with a metallic interruption of length l_{short} that is assumed to be sufficiently large, so that the magnetic current induced in the slot for $x > d + l_{\text{short}}$ does not influence the current at $x < d$. This assumption allows modeling a semi-infinite slot with infinitely extended metal (for $x > d$) as an infinite slot with a finite metal termination. Such approximation is convenient due to the availability of the infinite slot spectral Green's function [4]. To solve for the unknown equivalent magnetic current on the slot, we introduce two basis functions as shown in Fig. 1(b): one represents the uniform current distribution excited in the gap, $f_\delta(x)$, while the other represents the edge-singular electric current induced at the shorted end, $f_g(x)$. The basis functions can be written in the spectral domain as



(a) (b)

Fig. 1. (a) Interrupted infinite slot printed on a dielectric slab. (b) Space domain basis functions with respect to their location along the semi-infinite slot.

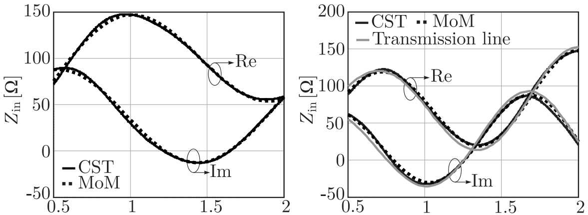
$$F_\delta(k_x) = \text{sinc}(k_x \delta/2) \tag{1}$$

$$F_g(k_x) = e^{jk_x g/2} \times \left(J_0\left(\frac{k_x g}{2}\right) - j\mathbf{H}_0\left(\frac{k_x g}{2}\right) - \frac{2}{\pi}\text{sinc}\left(\frac{k_x g}{4}\right) e^{-jk_x g/4} \right) \tag{2}$$

where $\mathbf{H}_0$ is the zeroth order Struve function and k_x is the spectral counterpart of the spatial variable x. The parameter g in (2) is related to the width of the current distribution on the metallic interruption. The value of g was found empirically to be linked to the width of the slot and the free-space wavelength as $g = (5/3)\sqrt{w\lambda}$.

The self and mutual impedances are calculated with the following spectral integrals

$$Z_{j,i} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{F_i(k_x) F_j(-k_x) e^{jk_x(x_i - x_j)}}{D_s(k_x)} dk_x \tag{3}$$



(a) (b)

Fig. 2. Comparison between the input impedance of a semi-infinite slot calculated with our method and CST. The geometrical parameters of the structure are $d = \lambda_0/4$, $w = \lambda_0/50$ and $\delta = \lambda_0/40$. λ_0 is the wavelength in free space at f_0. (a) The slot is surrounded by free space. (b) A thin dielectric substrate is added: $\varepsilon_r = 4$, $h = \lambda_d/20$ when λ_d is the wavelength in the dielectric at f_0.

where the subscripts i and j are either δ or g and $x_\delta = 0$ and $x_g = d$. $D_s(k_x)$ is the spectral longitudinal Green's function of an infinite slot, defined in [4].

Figure 2 shows the input impedance of a semi-infinite slot, both in free space and in the presence of a thin dielectric slab, calculated using our method, compared to CST.

III. EQUIVALENT TRANSMISSION LINE MODEL

The integrand in (3) presents two types of singularities: square-root branch points representing the space waves radiating away from the slot, and poles associated with quasi-TEM waves launched along the slot. In the presence of a thin dielectric substrate, the pole singularity does not coincide with the branch point in the complex k_x-plane, so that the polar contribution can be isolated. The location of the pole, k_{xp}, can be found using a local-search algorithm starting from k_0. Using Cauchy's theorem the polar contributions to the mutual impedances are evaluated and they can be represented as the equivalent transmission line circuit shown in Fig. 3.

The characteristic impedance of the transmission line is found as in [5] to be $Z_{0,s} = -2j/D'_s(k_{xp})$, where the prime ($'$) indicates the operation of differentiation.

The turn ratios of the two transformers are $n_\delta = F_\delta(-k_{xp})$ and $n_g = F_g(-k_{xp})$. The self impedances are split into the contributions of the transmission line and remaining terms

$$Z_{ii,\text{rem}} = Z_{ii} - n_i^2 \frac{1}{2} Z_{0,s} \tag{4}$$

where the subscript i is either δ or g. We define a single impedance to represent the end-point of the semi-infinite slot:

$$Z_{\text{end}} = \frac{\left(Z_{gg,\text{rem}}/n_g^2\right) Z_{0,s}}{\left(Z_{gg,\text{rem}}/n_g^2\right) + Z_{0,s}}. \tag{5}$$

For small distances d the accuracy of the transmission line model decreases, since the interaction between the two basis functions is not described by the mode only propagating along the slot, but also by the space wave coupling, which is not accounted for in the model.

The result of the transmission line model is compared with the MoM solution in Fig. 2(b).

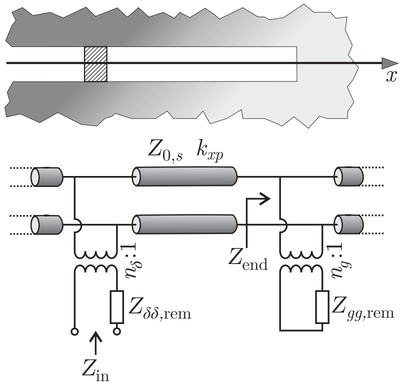


Fig. 3. Equivalent transmission line circuit representing the two basis functions of the semi-infinite slot.

IV. CONCLUSION

An efficient method of moments solution for planar slots embedded in generic stratified media was presented, with only two basis functions, one located at the feeding point and one at the termination of the slot. The basis functions were chosen such that they properly account for the reactive energy localized at these points.

Based on the numerical solution, an equivalent transmission line circuit was derived. The radiation is described in the model as resistances located at the feed and the end point. This approach allows representing the radiation from the slot as the generation of different space waves, one associated with the feeding gap and one emerging from the end point. The physical dimensions and the shape of the basis functions was accounted for in the circuit by means of transformers.

The method can be used to describe finite slots by terminating the slot on both sides of the feed.

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