Reflector Antenna Optimisation with Multi-level Coordinate Search

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***Abstract*—We review the key aspects of an optimisation algorithm when applied to challenging antenna design problems, and highlight the Multi-level Coordinate Search (MCS) global optimisation algorithm as an option in some scenarios. We present two comparisons which illustrate that MCS can be beneficial compared to a local algorithm as well as other popular global algorithms.**

I. Introduction

When designing antennas for modern communication systems, optimisation using mathematical optimisation algorithms is an almost ubiquitous part of the workflow. When choosing between different optimisation algorithms, there are a number of choices engineers need to make in order to balance computational resources with the achieved performance of the resulting optimised system.

In this paper, we discuss some of the considerations that should be behind a choice of optimisation algorithm, and then highlight one algorithm which can be appealing in some cases, the Multi-level Coordinate Search [1].

The optimisation problem that is solved in the present paper can be expressed as computing the solution $\boldsymbol{x}^\star$ of the following mathematical problem

$$\boldsymbol{x}^\star = \arg\min_{\boldsymbol{x}} \quad \max\left(\boldsymbol{r}(\boldsymbol{x})\right) \qquad \text{s.t.} \quad \boldsymbol{l} \leq \boldsymbol{x} \leq \boldsymbol{u} \tag{1}$$

In words, we seek the set of N parameter values $\boldsymbol{x}^\star$ that minimizes the maximum of M residuals $\boldsymbol{r}(\boldsymbol{x}^\star)$. The parameter values have to fullfil the condition that they are larger than or equal to the corresponding elements in the vector $\boldsymbol{l}$ and smaller than or equal to the corresponding elemeents in $\boldsymbol{u}$.

II. Algorithm considerations

When faced with a specific optimisation problem, the engineer should among other things consider the following aspects:

- Will a local optimum be sufficient, or is a global optimum necessary, in spite of the often greatly increased computational effort?
- Is the quality of the starting guess good enough to trust that a sufficiently good optimum can be found by a local algorithm?

Several local algorithms for (1) exist, both general-purpose algorithms such as Nelder-Mead and BFGS-type methods as well as custom-tailored algorithms for the Min-Max formulation such as [2]. However, if a global optimum is deemed worthwhile, or if a good starting guess cannot be found, those algorithms will generally not provide sufficient performance, and global algorithms must be considered — if the number of variables is modest, say, $N \lesssim 10$.

A very large number of global optimisation algorithms exists. In the mathematical literature, the main discerning feature between global algorithms is the balance between exploration (the tendency to explore the entire domain of the function) and exploitation (the aggresiveness when finding something that looks like a local minimum). Algorithms like Genetic Algorithms, Simulated Annealing e.t.c., lean towards exploration which carries with it a promise of avoiding local minima, but also gives a high number of function evaluations.

III. Multi-level Coordinate Search

In this paper, we will demonstrate the capabilities of the Multi-level Coordinate Search (MCS) [1] algorithm. The algorithm is a method of combining heuristics along each variable that is being optimised, in order to act as a preprocessor for a local optimisation algorithm. Thus, MCS attempts to find one or more points that seem to be good places from where to start a local optimisation, whilst ensuring that the domain of the function is reasonably well explored. By applying this methodology, MCS leads to a slightly poorer exploration than e.g. Genetic Algorithms, but is still able to provide good candidates for local algorithms at a fraction of the computational resources required by most other global algorithms.

MCS works by conducting a series of so-called ”sweeps”, each of which perform a hierarchical partitioning (the ”Multi-level” part of the MCS name) of the domain specified by the lower and upper bounds $\boldsymbol{l}$ and $\boldsymbol{u}$. The partioning is performed along the coordinate axis, and the decision on which part of the domain to partition is made based on quadratic interpolation models along the axis, as well as a set of heuristics that indicate where to expect the greatest function improvement. At the end of each such sweep, if deemed relevant by the MCS algorithm, a local search is started using any local algorithm. In our case, we generally find that such local searches are started close to a good optimum, and therefore we choose an accurate optimisation algorithm such as the Min-Max described in [3], or a simple derivative-free algorithm based on [4] with some modifications, as the local algorithm.

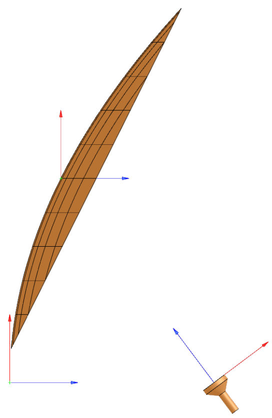

Fig. 1. The initial configuration of the antenna system.

TABLE I
RESULTS FROM THE OPTIMISATION ALGORITHMS FOR CASE A, SCANNING $4°$. THE STARTING DIRECTIVITY AT $\theta = 4°$ IS 11.12 DBI.

	Min-Max	MCS	CMA-ES	GA
Evaluations	23	123	501	140
Directivity at $\theta = 4°$ [dBi]	11.35	38.77	25.02	14.04

IV. RESULTS

To set up a simple test case, we consider a reflector system design, where the beam needs to be scanned slightly relative to a simple canonical system.

The initial configuration is shown in Figure 1. It consists of an offset circular parabolic reflector with a diameter of $D = 1\,\mathrm{m}$. The feed is a corrugated horn operating at $f = 12\,\mathrm{GHz}$, placed $0.6\,\mathrm{m}$ away from the reflector along the rotation axis of the system, and the clearance between feed and the bottom edge of the reflector is $0.1\,\mathrm{m}$. The feed is simulated by applying the Method of Moments add-on in TICRAs software GRASP [5], and the reflector is simulated using Physical Optics (PO) augmented by the Physical Theory of Diffraction (PTD) - the reflector could also be simulated using full-wave methods, but is not necessary due to the accuracy of the PO/PTD implementation in GRASP.

A. Four degrees off-axis

We now demand that the peak directivity of the system is moved $4°$ off-axis by applying the optimisation methods implemented in GRASP, and as a first case, we allow the optimiser to vary the position of the feed in the focal plane, attempting to maximise the directivity at $4°$ off-axis. We compare the performance of the local Min-Max algorithm in GRASP [3] with the performance of MCS with a derivative-free algorithm as the local algorithm. The comparison is shown in Table I.

TABLE II
RESULTS FROM THE OPTIMISATION ALGORITHMS FOR CASE B, SCANNING $8°$. THE STARTING DIRECTIVITY AT $\theta = 8°$ IS -11.82 DBI.

	Min-Max	MCS	CMA-ES	GA
Evaluations	44	359	1001	310
Directivity at $\theta = 8°$ [dBi]	-1.94	36.42	33.44	15.56

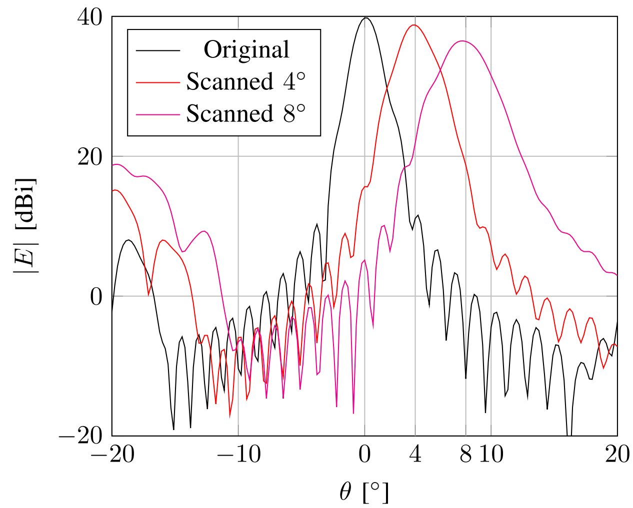

Fig. 2. The beam for the original system showed in black, along with the scanned beams.

As the table shows, the Min-Max algorithm rapidly reaches a local minimum, as one of the side-lobes of the nominal pattern is near the $4°$ direction. For MCS, a much better result is achieved, succesfully moving the peak of the pattern. The resulting original and scanned pattern is shown in Figure 2. The performance of MCS is also better than two other popular global solvers, the Covariance Matrix Adaptation Evolution Strategy (CMA-ES) and an implementation of a Genetic Algorithm (GA). The scan loss is about 1 dBi.

B. Eight degrees off-axis

We then move on with a second example, tilting the beam further, asking for peak directivity $8°$ off-axis. Here, the original beam directivity is -11.82 dBi, indicating a near-null value as can be seen in the black curve in Figure 2. As optimisation variables, we allow the feed to be moved in the focal plane as before, but also allow rotation of the feed in all three axis, for a total of 5 variables.

We then again compare the performance of the Min-Max algorithm with the performance of MCS. The comparison is shown in Table II. This time, Min-Max achieves a fair improvement of about 10 dBi relative to the starting position, but does not succeed in rotating the main beam. GA also fails, and CMA-ES manages a reasonable result, albeit requiring 1001 function evaluations. MCS, however, succeeds in scanning the beam the full $8°$ and requires only 359 evaluations.

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