Propagation in a Temporally Modulated Transmission Line: Exotic Band Structures and Reconfigurable Applications

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In the first part of this talk we will briefly review wave propagation in systems with different types of periodicity: purely spatial[1], purely temporal[2], space-time[3], and (independently) spatial *and* temporal[4]. The second part of the talk is dedicated to novel results for the last-mentioned kind of periodicity[4]. Specifically, we derive a complex electromagnetic band structure for propagation in a discrete transmission line with capacitors that are periodically modulated in time.

In the case of space-time periodicity[3] a characteristic parameter (permittivity, capacity, etc) is a periodic function of the phase of a large-amplitude "pump wave"; the period is, simply, 2π. On the other hand, for spatial and temporal periodicity a characteristic parameter is independently periodic in both space and time, with periods given, respectively, by a lattice constant a (in 1D) and a period $T = 2\pi/\Omega$, where Ω is the angular modulation frequency. Our group has recently reported the first experimental observation of a ("vertical") band-gap in the phase advance ka for a discrete, modulated, low-pass transmission line[4]. Here, the object of our investigation is a lumped band-pass transmission line with the capacitors ("varactors") in all the unit cells being modulated in tandem (simultaneously). What kind of waves can propagate in such a doubly periodic system?

We assume that the same harmonic modulation voltage $V_\Omega(t)$ is applied to every varactor of the infinitely long transmission line. In addition, we have a propagating voltage wave $v_N(t)$ of small-amplitude, N being the serial number of the unit cell. Then the total voltage on the N'th varactor is

$$V_N(t) = V_\Omega(t) + v_N(t) = \bar{V}_\Omega[1 + m\sin(\Omega t)] + v(t)e^{ikaN} \quad (1)$$

Here, m is the strength of modulation, $\bar{V}_\Omega$ is the average modulation voltage, ka is the phase advance over a single unit cell, and $v(t)$ is the time-dependent amplitude of the wave, where $|v(t)| << \bar{V}_\Omega$. Because the capacitances are periodic in time, $v(t)$ must satisfy the Floquet-Bloch theorem

$$v(t) = \tilde{v}(t)e^{-i\omega t} = \sum_r \tilde{v}_r(\omega)e^{-i(\omega - r\Omega)t} \quad (2)$$

where ω is the signal frequency and the summation is over all

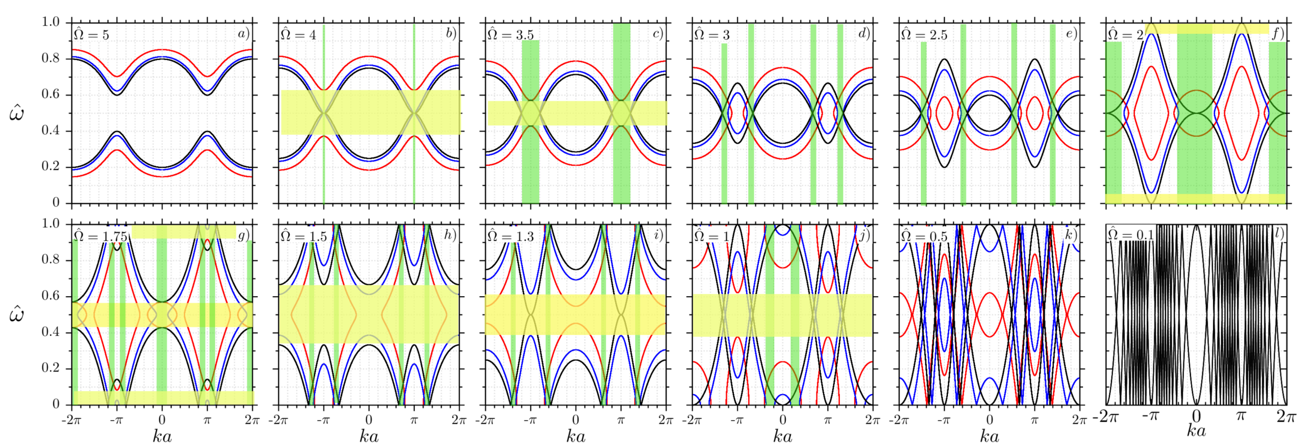

Fig. 1: Electromagnetic band structures $\hat{\omega}(ka)$ for 12 values of the reduced modulation frequency $\hat{\Omega} = \Omega/\omega_0$ from $\hat{\Omega} = 5$ in (a) to $\hat{\Omega} = 0.1$ in (l). The modulation m takes the values 0 ("empty temporal lattice", black lines), 0.2 ("weak modulation", blue lines), and 0.5 ("strong modulation", red lines).

the harmonics $r\Omega$ $(r = ... - 1, 0, 1, ...)$ of the modulation frequency. Namely, the wave $v_N(t)$ is a superposition of harmonic waves of frequencies ..., ω, $\omega \pm \Omega$, $\omega \pm 2\Omega$, ... Then, Kirchhoff's voltage and current laws, applied to a unit cell, lead to an eigenvalue equation for the Fourier coefficients $\tilde{v}_r(\omega)$ in eq.(2). The solution of the eigenvalue problem yields the relation between ω and ka, namely the electromagnetic band structure. This is shown in Fig.1 for a series of reduced modulation frequencies $\hat{\Omega} = \Omega/\omega_0$ (where ω_0 is the resonant frequency of a unit cell) and, for each of these, three values of the modulation strength m. Only one period in the reduced frequency $\hat{\omega} = \omega/\Omega$ $(0 < \hat{\omega} < 1)$ and two periods in the phase advance $(-2\pi < ka < 2\pi)$ are shown; it is understood that these are repeated infinitely in the vertical and horizontal directions. Even in the limit of no modulation $m = 0$ ("empty temporal lattice" model, black lines) large frequency gaps are present - simply because the pass-bands are narrow. The band structures $\hat{\omega}(ka)$ for $m = 0.2$ ("weak modulation", blue lines) are quite similar to those for $m = 0$, however a new feature emerges, namely, prohibited bands of the phase advance ka are produced. For $m = 0.5$ ("strong modulation", red lines) further qualitative changes occur, with the appearence of both ω- and k-gaps. It is important to note that a change in $\hat{\Omega}$ or in m can generate forbidden frequency gaps (yellow horizontal bands) and forbidden propagation-constant gaps (green vertical bands). All the four logical possibilities can be observed in Fig.1: only ω-gap, only k-gap, simultaneous ω- and k-gaps, and no gap at all.

The discrete transmission line, with capacitors that are periodically modulated in time, is convenient for studying wave propagation in systems with periodicity both in space and in time. The band structure for electromagnetic waves that can propagate in such a modulated transmission line depends qualitatively on the two easily controllable parameters $\hat{\Omega}$ and m. This variability could lead to reconfigurable applications for filters, frequency multipliers, delay transmission lines, and metamaterial properties. Analogously, propagation of other kinds of waves in other kinds of systems that are independently periodic in both space and time, could give rise to interesting band structures and useful applications.

References

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