Low-Rank- and Butterfly-Compressibility of Moment Matrix Blocks

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Fast direct integral-equation solvers for electromagnetic wave problems rely on the structured compression of the method-of-moments (or impedance) matrix. Any fast direct solver comprises two key components: (i) An integral equation that gives rise to matrices with blocks that are, in some sense, compressible. (ii) An algebraic (often hierarchical) procedure for computing, out of the compressed representations of these blocks, a compressed factorized (essentially solved) representation of the entire matrix. Early works on direct solvers focused on exploiting the rank-deficiency of off-diagonal matrix blocks, which correspond to low-rank (LR) interactions between non-overlapping (and often separated) subdomains of the analyzed geometry, for their compression via LR-approximation. For topologies and integral-equation kernels yielding interactions that are either of actual (E. Corona, P.G. Martinsson, and D. Zorin, Appl. Comput. Harmon. Anal., 38 ,284-317, 2015) or effective (Y. Brick, V. Lomakin, and A. Boag, IEEE Trans. Antennas Propag., 62, 4314–4324, 2014) reduced dimensionality, LR compression leads to reduced asymptotic complexity of fast direct solvers. For general topologies, which yield inherently LR-incompressible blocks, savings in memory and computation time are still exhibited, for rather large problems. However, as the rank approaches its asymptotic scaling with subdomain size, it increases proportionally to the number of problem unknowns, such that the asymptotic complexity is not reduced. A recently proposed approach suggests replacing the LR-representation by its hierarchical "butterfly" extension (H. Guo, Y. Liu, J. Hu, and E. Michielssen, IEEE Trans. Antennas Propag., 65, 4742–4750, 2017). It has been shown that the resulting blocks of the factorized matrix inherit the butterfly-compressibility of the impedance matrix blocks for general topologies, leading to superior asymptotic complexity to that obtained with LR compression. Still, the relatively simpler operation of LR-approximations and their very good performance in the pre-asymptotic regime may suggest that the computation time and memory savings obtained via butterfly compression can be further optimized, if the scheme's parameters are carefully set. Specifically, with the butterfly factorization being a multilevel extension of the LR-approximation, the depth of this hierarchical structure may be tuned for improved compression and overall solver performance.

In this work, we will study the influence of butterfly depth on the compression of impedance matrix blocks. To that end, we would like to be able to compare the memory savings obtained with various butterfly depths, for various values of the prescribed compression accuracy. The comparison should be performed for large source and observer domains. For the lower end of butterfly depth values, we will need to compute the LR-approximation of blocks representing interactions between rather large bottom-level subdomains and the observer. These expensive computations will be accelerated by employing fast LR-approximation algorithms (Y. Brick and A. E. Yılmaz, IEEE Trans. Antennas Propag., 66, 5359–5369 ,2018). Using the bottom-level rank-revealing analysis, we will also compute a preliminary estimate, via extrapolation, of the memory required for storing the entire butterfly-approximation. From the curves describing the memory as a function of butterfly depth, we will deduce guidelines for selecting the butterfly depth, given the subdomains topology, integral equation kernel, and mesh density.