Water-Table Detection in a Hyper-Arid Region

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***Abstract*—A finite-difference time-domain (FDTD) method, accelerated by imposing surface impedance boundary condition (SIBC), is implemented to compute the scattering fields from a hyper-arid soil above an aquifer. An estimation method of water-table depth beneath ground surface is proposed, which is based on the interference pattern of backscattering fields at normal incidence.**

Keywords—FDTD, surface impedance boundary (SIBC), random medium, bistatic scattering coefficient (BSC), water table.

I. INTRODUCTION

Modern airborne or satellite-borne radars can cover large areas at sufficiently high spatial and temporal resolutions. Bistatic scattering coefficients (BSCs) were used to retrieve soil moisture [1], [2]. The aquifer and vadose zone in a hyper-arid region reveal high permittivity contrast, leading to interference pattern of BSC versus water-table depth.

Finite-difference time-domain (FDTD) method is flexible in simulating scattering fields from an inhomogeneous medum [3]. The FDTD method takes tremendous computational time to simulate a very lossy random medium at fine spatial resolution. By imposing a surface impedance boundary condition (SIBC) on the surface of a large-volume medium may significantly reduce the computational time. The concept of time-domain surface impedance received attention around 1990s. Beggs *et al*. introduced a constant SIBC which is applicable at a single frequency as well as a dispersive SIBC which is applicable over a wide frequency band and a wide range of conductivity [4]. A frequency-dispersive SIBC is equivalent to a convolutional type of time-domain SIBC. The former can be approximated as a rational function. Maloney and Smith proposed a more general method amended by preprocessing [5].

In this work, an FDTD method in conjunction with a constant SIBC is proposed to compute the scattering fields from hyper-arid soil above aquifer, and the depth of water table beneath ground is estimated by using the interference pattern of the BSC.

II. IMPLEMENTATION OF TIME-DOMAIN SIBC

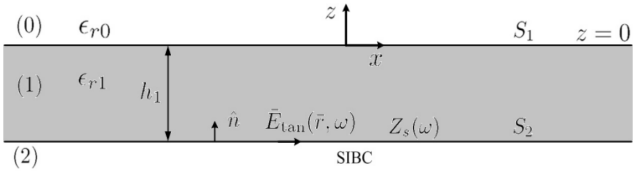

Fig. 1. Scattering from a two-layered medium, layer (0) is air, layer (1) is vadose zone with thickness h_1 , layer (2) is aquifer, and SIBC is imposed at the interface S_2 between layers (1) and (2).

Fig.1 shows the scenario of scattering from hyper-arid soil above an aquifer, which is modeled as a two-layered medium with SIBC at the interface S_2 between layers (1) and (2). Consider a time-harmonic uniform plane wave (UPW) propagating downwards S_1 , at an incident angle θ_0 . The SIBC at the interface S_2 can be represented as [6]

$$\bar{E}_{\tan}(\bar{r},\omega) = Z_s(\omega)\hat{n}\times\bar{H}(\bar{r},\omega)$$

where $\bar{E}_{\tan}(\bar{r},\omega) = \bar{E}(\bar{r},\omega) - \hat{n}\hat{n}\cdot\bar{E}(\bar{r},\omega)$ is the tangential electric field and $Z_s(\omega) = \eta_0/\sqrt{\epsilon_{r2}}$ is the surface impedance, η_0 is the intrinsic impedance in free space, and $\epsilon_{r2} = \epsilon'_{r2} - j\sigma_2/(\omega\epsilon_0)$ is the relative permittivity of layer (2). The updating equation in the FDTD scheme reads

$$\bar{E}^n_{\tan} = \frac{\eta_0}{\sqrt{\epsilon'_{r2}}}\left[\hat{n}\times\bar{H}^n + \sum_{m=0}^{n-1} F^m\left(\hat{n}\times\bar{H}^{n-m-1/2}\right)\right]$$

with

$$\begin{aligned} F^m &= \int_m^{m+1} a\Delta t e^{au\Delta t}\left[I_1(au\Delta t) + I_0(au\Delta t)\right]du \\ &= e^{a(m+1)\Delta t} I_0(a(m+1)\Delta t) - e^{am\Delta t} I_0(am\Delta t) \end{aligned}$$

where I_n is the modified *n*th order Bessel function of the first kind and $a = -\sigma_2/(2\epsilon'_2)$.

III. ESTIMATION METHOD OF WATER-TABLE DEPTH

The procedure to estimate the water-table depth is summarized below.

Step (1): Choose a reference two-layered medium and compute a reference curve of backscattering coefficient versus h_1/λ .

Step (2): Compare the measurement data with the reference curve to estimate the bias of the measurement apparatus.

Step (3): Apply the apparatus to measure unknown soil over a specific frequency band.

Step (4): Add the bias estimated in Step (2) to the field measurement data obtained in Step (3).

Step (5): Match the peak value in Step (4) to the closest peak value in the reference curve, then retrieve the h_1/λ value associated with the peak value.

Step (6): The frequency f_p associated with the peak value is used to estimate the depth as $h_1' = \frac{c}{f_p}\frac{h_1}{\lambda}\Big|_{f=f_p}$, where c is the speed of light in free space.

IV. Simulations and Discussions

The SIBC-FDTD method is applied to simulate wave scattering when a water table appears beneath a homogeneous and a random vadose zone, respectively. Fig.2 shows the time-domain waveform of backscattering field from a two-layered homogeneous soil, sampled at $P(0,0,0.75\lambda)$. Both waveforms match reasonably well. Pulses ① and ② are the reflected signals from S_1 and S_2, respectively. Both are 180° out of phase from the incident waveform. Pulse ③ transmits through S_1, then is reflected in sequence by S_2 (reverse sign), S_1 and S_2 (reverse sign again), and finally transmits through S_1 to reach P. Since pulse ③ is reversed twice in sign, it becomes in phase with the incident waveform. The amplitude of pulse ① is smaller than that of pulse ②, indicating that the reflection coefficient at S_2 is stronger than that at S_1 since layer (2) is much denser in permittivity than layer (1).

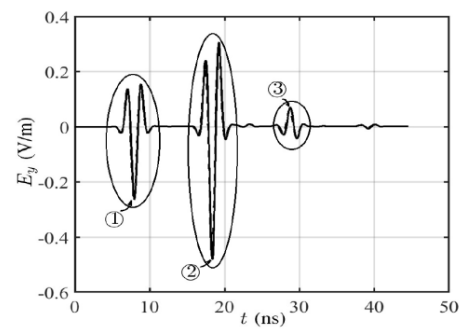

Fig. 2. Time-domain waveform of backscattering field from a two-layered soil, sampled at $P(0,0,0.75\lambda)$, with $h_1 = 1.5\lambda$, $\varepsilon_{r1} = 3$, $\varepsilon_{r2} = 30 - j6.607$, f = 500 MHz, $L_x = L_y = 8\lambda$, $\theta_i = 0^\circ$, —: SIBC-FDTD, ---: conventional FDTD.

Fig.3 shows an example of applying the proposed procedure to determine h_1 by using the peak value of backscattering coefficient from a random vadose zone, in which the electric properties are presumed known. To demonstrate the working principle, a water-table depth of h_1 = 60 m is chosen to generate the backscattering coefficients at several different frequencies. These backscattering coefficients are then calibrated by adding the bias which was previously determined. These data are then matched with the curve of backscattering coefficient versus h_1/λ,

$$\gamma_{hh} = \frac{L_x L_y k_0^2}{\pi}\left|R_{\cap 0}^{\mathrm{TE}}\right|^2 \qquad (1)$$

where $R_{\cap 0}^{\mathrm{TE}}$ is the total reflection coefficient at S_1. In this example, two peak values are matched at frequencies of 10.5 MHz and 12 MHz, corresponding to h_1/λ = 2.1 and 2.4, respectively. Thus, the water-table depth is estimated to be h_1' = 60 m, which matches the reference depth. The sensitivity study is then conducted by simulations.

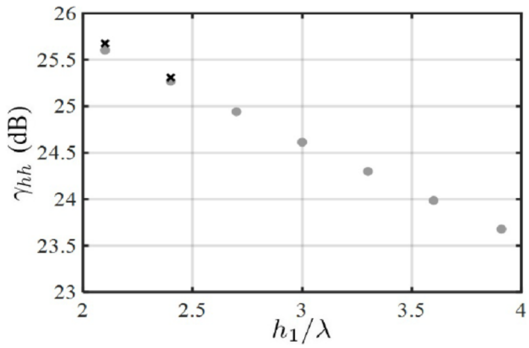

Fig. 3. Peak value of backscattering coefficient from a random vadose zone versus h_1/λ, with h_1 = 60 m, grey •: peak value by using (1), ×: peak value by using SIBC-FDTD.

V. Conclusion

An FDTD method, accelerated by imposing surface impedance boundary condition (SIBC), is proposed to compute the scattering fields from hyper-arid soil above aquifer. An estimation procedure of water-table depth in a hyper-arid region is proposed, based on the backscattering coefficient at normal incidence over a specific frequency band.

References

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