A Partially Coherent Approach for Scattering of Electromagnetic Waves from Random Layered Media with 3D Rough Interfaces

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***Abstract*—Statistical nature of the problem introduces a randomness in the phase of the electromagnetic field such that the fields are not coherent after traveling in the medium for a few correlation lengths. It is shown that for such a problem we can use a combined coherent-incoherent approach to greatly reduce the complexity of the problem.**

I. Introduction

Scattering of the Electromagnetic waves from the Layered media with random rough interfaces has many applications in broad areas of the science and engineering, in particular, we are interested in the multi-layered media with rough interfaces as a forward model of microwave remote sensing of the ice sheets in the Arctic and Antarctica. One of the signatures of ice sheets is that the permittivity (as well as thickness) of layers is fluctuating with height due to the accumulation patterns.

The first statistical factor in the problem is the randomness of the interfaces. For given dielectric constant profile along z, this problem can be solve in variety of the ways [2]. However, in order to calculate the averaged quantities we need to run a Monte-Carlo simulation over realizations of the surface profiles. If the specifications of the problem can fit into the Small Perturbation Method (SPM), it provides an analytical solution of the fields with lower amount of computation and also averaging process is performed in the formulation.

Apart from surface profile, the dielectric constant along z is also random. Therefore, a Monte-Carlo over dielectric profile realization is required. Assume that the number of layers (N) in the problem is very large, then the number of required realizations to get a convergent solution would be very large (see III). We can utilize this randomness to facilitate the computation of the problem [1]. Since the dielectric profile has a finite correlation length along z, wave inside the layer media will lost it's phase coherency after traveling beyond the correlation length of the process $\varepsilon(z)$. If we divide the whole media into blocks (Fig. 1) larger than the correlation length of $\varepsilon(z)$, within each block fields have some degree of coherency and it must be solve exactly by employing SPM (which exactly accounts for phase of the fields). We can cascade the adjacent blocks through their intensities rather than field amplitudes. This combined coherent-incoherent approach greatly reduce number of required realizations to get a convergent solution. Cascading equations is derived based on the power conservation. In this paper the cascading equations for specular and diffused intensities through the blocks are provided in section II. In section III, we consider a numerical example to show the agreement between exact (coherent) and partially coherent approaches.

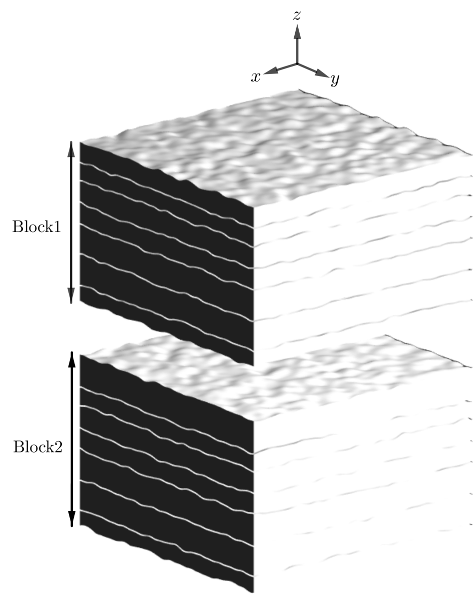

Fig. 1. Two blocks of layered media with rough interfaces.

II. Problem Formulation

Consider a block of length L_B which contains N_B layers with rough interfaces. For solution of the scattered and transmitted Poyinting vector $\overline{S}$ from SPM we have [3]

$$\frac{\langle \overline{S}_s^{\alpha} \cdot \hat{z} \rangle}{\overline{S}_{\mathrm{i}}^{\beta} \cdot (-\hat{z})} = \int_{(2\pi)^+} \mathrm{d}\Omega_s \; \gamma_{\alpha\beta}(\theta_s, \phi_s; \theta_i, \phi_i) \qquad (1)$$

Here, β and α are source and response polarizations, and $\gamma_{\alpha\beta}$ and $\xi_{\alpha\beta}$ are reflectivity and trasmissivity of the media, respectively. Reflectivity have two components, specular $\gamma_{\alpha\beta}^{\mathrm{coh}} \propto \delta_{\alpha\beta}\delta(\Omega_s - \Omega_i)$ and an incoherent component $\gamma_{\alpha\beta}^{\mathrm{inc}}$ which is responsible for angular broadening as well as polarization mixing of the intensities. For each block we need to know the reflectivity/transmissivity when the block is excited from up/down, for example, $\gamma_{1,u}$ is reflectivity of block 1 when excited from the top. For the problem of passive remote sensing, if we assume statistically isotropic surfaces, the response can be integrated along ϕ to find parameters that depends on only θ. If we assume the steady state upward and downward going intensities between blocks are given by $I_u^\alpha(\mu)$ and $I_d^\alpha(\mu)$, where α is polarization channel and $\mu = \cos\theta$, then different intensities are related together through

$$I_s^\alpha(\mu) = \sum_\beta \int_0^1 \mathrm{d}\mu' \Big[\xi_{1d}^{\alpha\beta}(\mu,\mu') I_u^\beta(\mu') + \gamma_{1u}^{\alpha\beta}(\mu,\mu') I_i^\beta(\mu')\Big]$$
$$I_d^\alpha(\mu) = \sum_\beta \int_0^1 \mathrm{d}\mu' \Big[\xi_{1u}^{\alpha\beta}(\mu,\mu') I_i^\beta(\mu') + \gamma_{1d}^{\alpha\beta}(\mu,\mu') I_u^\beta(\mu')\Big]$$
$$I_u^\alpha(\mu) = \sum_\beta \int_0^1 \mathrm{d}\mu' \; \gamma_{2u}^{\alpha\beta}(\mu,\mu') I_d^\beta(\mu')$$
$$I_t^\alpha(\mu) = \sum_\beta \int_0^1 \mathrm{d}\mu' \; \xi_{2u}^{\alpha\beta}(\mu,\mu') I_d^\beta(\mu') \qquad (2)$$

This is a system of coupled integral equations for I_d and I_u. In order to solve it we decompose each intensity into $I(\mu) = I^{(0)}(\mu) + I^{(1)}(\mu) + I^{(2)}(\mu)$ where the superscript shows total number of non-specular reflections and/or transmissions. This expansion accounts for up to the second order diffused scattering and other mechanisms can be neglected (as those are very small). The system of Eq. (2) can be solve by balancing orders of scattering. The coherent reflectivity of the equivalent block $\tilde{\gamma}_u^{\alpha,\mathrm{coh}}$ can be obtained as

$$\tilde{\gamma}_u^{\alpha,\mathrm{coh}} = \frac{I_s^{(0)\alpha}}{I_0^\alpha} = \xi_{1d}^{\alpha,\mathrm{coh}} \frac{\xi_{1u}^{\alpha,\mathrm{coh}} \gamma_{2u}^{\alpha,\mathrm{coh}}}{1 - \gamma_{1d}^{\alpha,\mathrm{coh}} \gamma_{2u}^{\alpha,\mathrm{coh}}} + \gamma_{1u}^{\alpha,\mathrm{coh}} \qquad (3)$$

Similarly, for diffused reflectivity

$$\tilde{\gamma}_u^{\alpha\beta,\mathrm{inc}}(\mu,\mu_i) = \frac{I_s^{(1)\alpha}(\mu) + I_s^{(2)\alpha}(\mu)}{I_0^\beta(\mu_i)} \qquad (4)$$

III. Numerical Example

Consider two blocks, each of which includes $N_B = 30$ layers with average block length of $\overline{L}_B \approx 15\lambda_i$ with Gaussian dielectric profile with correlation length of $\ell_\varepsilon = \lambda_i \ll \overline{L}_B$. Each interface is Gaussian with correlation length of $\ell_s = 1.5\lambda_i$ and RMS height of $h = 0.02\lambda_i$. In order to evaluate the performance of the partially coherent approach, we compare combined solution of two blocks of 30 layers with the coherent response of the concatenated structure with 60 layers. A Monte-Carlo simulation over dielectric profiles is performed over 100 realizations for each block of 30 layers while concatenated block of 60 layers requires 900 realizations to converge.

Figures 2 and 3, plot the incoherent Co-pol and X-pol reflectivity of $\gamma_u^{ee,\mathrm{inc}}(\theta_s,\theta_i)$ and $\gamma_u^{he,\mathrm{inc}}(\theta_s,\theta_i)$. The coherent reflectivity comparison is given in Fig. 4 and they show a very good agreement between partially coherent (P.C) approach with the exact coherent (C) solution.

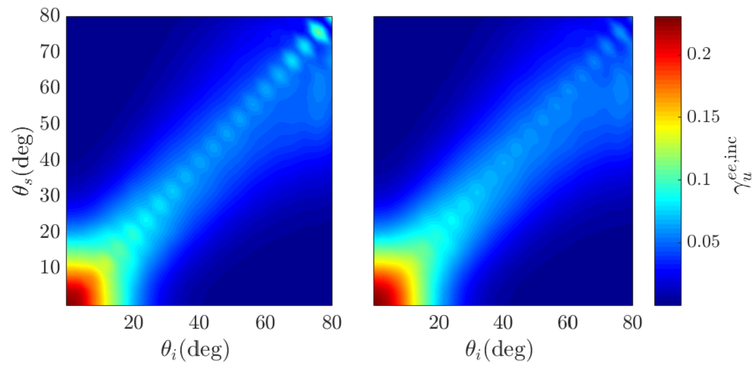

Fig. 2. Co-pol response of $\gamma_u^{ee,\mathrm{inc}}(\theta_s,\theta_i)$, right: P.C, left: C

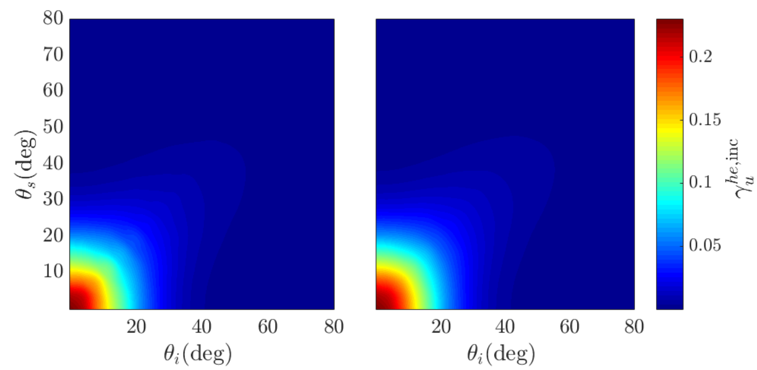

Fig. 3. X-pol response of $\gamma_u^{he,\mathrm{inc}}(\theta_s,\theta_i)$, right: P.C, left: C

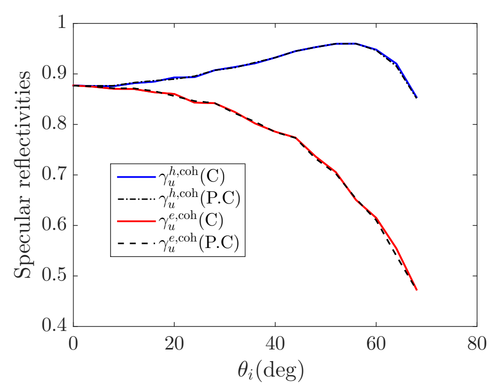

Fig. 4. Specular reflectivity obtained by P.C and C methods.

References

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[2] Sanamzadeh, Mohammadreza, Leung Tsang, and Joel Johnson. "3D Electromagnetic Scattering from Multi-Layer Dielectric media with 2D Random Rough Interfaces Using T-Matrix Approach." IEEE Transactions on Antennas and Propagation (2018).

[3] M.Sanamzadeh, L.Tsang , J.Johnson , R.Burkholder, S.Tan, *Scattering of electromagnetic waves from 3D multilayer random rough surfaces based on the second-order small perturbation method: energy conservation, reflectivity, and emissivity*. JOSA A, 2017, 34(3), p.395.