Uncertainty Evaluation of Rydberg Atom-based RF E-field Metrology

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***Abstract*—SI traceable radio frequency (RF) electric (E-) field measurement using a quantum coherence effect of Rydberg atoms can directly link the value to optical frequency and the Plancks constant, and this technique is promising for developing a broadband RF E-field sensor and a next-generation metrology standard. In this paper, a detailed measurement uncertainty budget is presented, and the major error sources and effects are comprehensively analyzed and assessed. The expanded uncertainty of 10.22 GHz E-field measurement in the range of 0.5 V/m to 2 V/m is 1.59 %.**

***Index Terms*—Uncertainty, Rydberg atoms, Quantum sensing, Metrology standard, Electric field**

I. INTRODUCTION

At present, radio frequency (RF) electric (E-) field measurements with a diode-based voltage detector cannot be traced to the SI units directly, cannot accurately detect fields below 1 V/m, and have uncertainties on the order of 0.5 dB or greater than 5 %. Presently used E-field probes were developed many decades ago and could be improved based on advances in quantum sensing techniques [1]. The Rydberg-atom-based SI traceable E-field measurement using electromagnetic induced transparency (EIT) and Autler-Townes (AT) splitting has received extensive attention recently and is promising to become a new RF E-field measurement standard [2], [3]. Compared with conventional standards, this method has advantages of ultra-broadband measurement covering from ~100 MHz to THz [4], high sensitivity with a predicted shot noise limit less than 1 μV·cm^{-1}·Hz$^{-1/2}$ [5].

Uncertainty assessment is one of the key work to promote a technique to be implemented as a metrology standard. In this paper, the uncertainty evaluation model, together with brief description of error sources is presented. Experiments are performed to determinate the uncertainty values for 10.22 GHz E-field measurement in the range of 0.5 V/m to 2 V/m.

II. RYDBERG ATOM-BASED RF E-FIELD MEASUREMENT

Counter-propagating probe and coupling lasers excite atoms in a vapor cell from ground state to a specific Rydberg state. RF E-field coupling specific Rydberg states could cause quantum interference in the probe absorption spectroscopy and an AT splitting appears within EIT spectroscopy. By measuring the optical frequency of the splitting (Δf_0), the E-field strength $|E|$ can be determined from (1), where $\hbar$ is Plancks constant, and μ is the atomic dipole moment [2].

$$|E| = 2\pi\frac{\hbar}{\mu}\Delta f_0. \tag{1}$$

In our experiment, a probe laser (780.24 nm) resonates in the D2 transition of $5S_{1/2}$ (F=2)$\rightarrow$ $5P_{3/2}$(F'=3) of rubidium (^{87}Rb) atoms and a coupling laser (479.85 nm) scans around the resonant frequency of transition $5P_{3/2}$(F'=3)$\rightarrow$ $59D_{5/2}$. The incident E-field of 10.22 GHz couples the nearby Rydberg states of $59D_{5/2}$ and $60P_{3/2}$.

III. UNCERTAINTY EVALUATION

A. Evaluation model and error sources

Uncertainty sources can be grouped in two categories: quantum-based and RF- induced uncertainties, as shown in Table I, which is for 10.22 GHz E-field measurement. According to (1), one of the quantum-based uncertainties contributing to the field strength determination is the calculation of atomic dipole moment (μ), and the results from different calculation models can be controlled within a relative variation of 0.1 % [3]. This is considered as a type B uncertainty. As shown in Table I, many effects have influence on the determination of AT splitting Δf_0. For example, the environment magnetic field of 1 Guass corresponds to Zeeman shift of 1~2 MHz; Polarization mismatch of E-field and laser field induces the variation of probe transmission spectroscopy [6]. The nonlinearity effect has been addressed in [3], and can be controlled within 0.82 % in the field strength range of 1 V/m to 2 V/m for this 10.22 GHz measurement.

B. Uncertainty assessment examples

1) Broadening mechanisms: Due to the homogeneous dephasing effects caused by a variety of broadening mechanisms [7], such as Doppler broadening, transit time broadening and power broadening, the lineshape of probe transmission spectroscopy can be affected. Figure 1 shows the effect of the vapor cell temperature on the ratio of EIT/AT peak height (H) over its full-width-half-maximum (FWHM), as a function of different probe and coupling laser power.

There is no buffer gas inside our vapor cell. The linewidth is dominated by collisions of atoms with the cell walls and

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TABLE I

THE MEASUREMENT UNCERTAINTY EVALUATION BUDGET.

Sources of Uncertainty		Description	Values $s(\bar{x}_i)$	Distribution	Divisor	c_i	$u(\bar{x}_i)$	v_i or v_{eff}
Dipole moment calculation, $u(p)$	**(1) Calculation error (type B)**	Calculation deviation of different models	0.10%	Normal	1	1	0.10%	inf
AT Splitting Determination $u(\Delta f)$	**(2) Fitting error**	Lorentz fitting error of AT splitting	0.32%	Normal	1	1	0.32%	inf
	(3) Environment field	Geomagnetic field (~0.5 Gauss) induced Zeeman effect	0.64%	Normal	1	1	0.20%	9
	(4) Broadening mechanism	Temperature and laser intensity	1.69%	Rectangular	1.73	1	0.31%	9
	(5) Two-photon detuning	Relative detuning noise between coupling and probe lasers	0.66%	Normal	1	1	0.21%	9
	(6) Nonlinear effects	For very weak field and near-ionization effect at extremely strong field	0.82%	Rectangular	1.73	1	0.15%	9
	(7) Polarization mismatch	Polarization mismatch between lasers and E-field	0.76%	Rectangular	1.73	1	0.14%	9
	(8) Technical noise	Photo diode detector and selection of amplifer gain	0.75%	Rectangular	1.73	1	0.14%	9
	(9) Repeatability	Measurement repeatability	0.61%	Normal	1	1	0.19%	9
	Combined uncertainty (quantum effects)			**Normal**			**0.62%**	90
RF effects	**(10) Vapor cell disturbance**	RF cavity resonance and scattering effect of a atomic vapor cell change the magnitude, polarization and spatial distribution of incident RF	2.67%	Rectangular	1.73	1	0.49%	9
	Combined uncertainty (total), u_c			**Normal**			**0.79%**	50
	Expanded uncertainty, $U_{95}(k=2.01)$			**Normal**			**1.59%**	

the spin-exchange dephasing, which is due to mutual collisions between atoms. The linewidth broadens as temperature increasing. The atomic density can be increased by heating the vapor cell, the peak height of the EIT/AT signal gradually rises and reaches a maxima. The optical dense effect reduces the actual laser power acting on the atomic gas, thus the peak height decreases when temperature goes even higher. As shown in Figure 1(a), the ratio of H/FWHM is maximized at about 60 °C. The vapor cell temperature is stabilized at this working point, and the variation of Δf_0 in temperature range of 40 °C to 60 °C is measured to determine the uncertainty.

2) Vapor cell disturbance: The vapor cell disturbance has been comprehensively investigated [6], [8]. To minimize field distortion, the vapor cell should be as small as possible (compared with RF wavelength), as thin as possible, made of a low-permittivity material, and the middle of the cell in direction of E-field vector is the position with least depolarization for field sensing. We use a 9-mm cubic vapor cell with wall thickness of 0.2mm to detect 10.22 GHz RF E-field. The field distribution normalized to the incident E-field strength is presented in Figure 1(b), The average deviation between measurement and incident field strength is about 2.67 %.

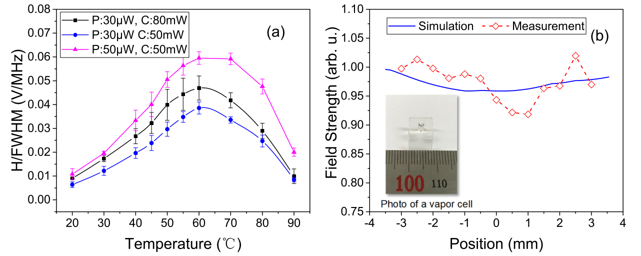

Fig. 1. Measurement optimization and uncertainty evaluation (a) Ratio of EIT/AT peak height to linewidth (H/FWHM) versus vapor cell temperature; (b)The optimization of field distortion induced by the vapor cell, indicating by internal field distribution of a 9-mm vapor cell.

IV. CONCLUSION AND FUTURE WORK

Uncertainty budget for Rydberg-atom-based E-field measurement is presented. As an example, the expanded uncertainty of 10.22 GHz E-field measurement in the range of 0.5 V/m to 2 V/m is 1.59 %. This work is valuable for understanding measurement errors and system optimization, can further promote this to be a metrology standard. In future publications we will address detailed uncertainty analysis and discuss how to control these effects.

REFERENCES

[1] C. Degen, F. Reinhard, and P. Cappellaro, “Quantum sensing,” *Rev. Mod. Phys.*, vol. 89, no. 3, Jul 2017.

[2] J. A. Sedlacek, A. Schwettmann, H. Kbler, R. Lw, T. Pfau, and J. P. Shaffer, Microwave electrometry with Rydberg atoms in a vapour cell using bright atomic resonances, *Nat. Phys.*, vol. 8, no. 11, pp. 819824, 2012.

[3] C. L. Holloway, M. T. Simons, J. A. Gordon, A. Dienstfrey, D. A. Anderson, and G. Raithel, “Electric field metrology for SI traceability: Systematic measurement uncertainties in electromagnetically induced transparency in atomic vapor,” *J. Appl. Phys.* , vol. 121, no. 23, p. 233106, Jun 2017.

[4] C. L. Holloway, J. A. Gordon, S. Jefferts, A. Schwarzkopf, D. A. Anderson, S. A. Miller, N. Thaicharoen, and G. Raithel, Broadband Rydberg atom-based electric-field probe for SI-traceable, self-calibrated measurements, *IEEE Trans. Antennas Propag.*, vol. 62, no. 12, pp. 61696182, 2014.

[5] H. Fan, S. Kumar, J. Sedlacek, H. Kbler, S. Karimkashi, and J. P. Shaffer, Atom based RF electric field sensing, *J. Phys. B At. Mol. Opt. Phys.*, vol. 48, no. 20, p. 202001, Oct. 2015.

[6] Z. Song, W. Zhang, Q. Wu, H. Mu, X. Liu, L. Zhang, and J. Qu, Field Distortion and Optimization of a Vapor Cell in Rydberg Atom-Based Radio-Frequency Electric Field Measurement, *Sensors*, vol. 18, no. 10, p. 3205, Sep. 2018.

[7] R. Boudot, P. Dziuban, M. Hasegawa, R. K. Chutani, S. Galliou, V. Giordano, and C. Gorecki, Coherent population trapping resonances in Cs-Ne vapor microcells for miniature clocks applications, *J. Appl. Phys.*, vol. 109, no. 1, 2011.

[8] H. Fan, S. Kumar, J. Sheng, J.P. Shaffer, C.L. Holloway, and J.A. Gordon, Effect of Vapor-Cell Geometry on Rydberg-Atom-Based Measurements of Radio-Frequency Electric Fields, *Phys. Rev. Appl.*, vol. 4, no. 4, p. 044015, Oct. 2015.