Recent Developments to the Discontinuous Galerkin Time-Domain Method for 3-D Multiscale Problems

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***Abstract*— Recent developments to the discontinuous Galerkin time-domain (DGTD) method for 3-D multiscale problems are reported in this paper. Although the DGTD method is very popular to electromagnetic problems at present, realistic electromagnetic wave propagation problems are often multiscale due to complex geometries or heterogeneous media, which leads to many restrictions of the traditional DGTD methods. Therefore, this paper reports on some significant advances about the DGTD method. First of all, in order to overcome the severe stability restrictions caused by the locally refined meshes, we propose a time integration strategy by combining excellent stability properties with a new explicit time scheme. Second, we apply this strategy into the inhomogeneous media to solve the multiscale dispersive problems. Considering some multiscale meshes with very small size, an unconditional stable hybridizable discontinuous Galerkin time method is proposed to increase time step so that greatly reducing computational time. Particularly, from meshes point of view, a new strategy is proposed by combining the DGTD with Multiscale Hybrid Method (MHM), and the parallel technologies can be greatly performed. By using the above methods, accurate numerical results can be obtained as well as a higher computational performance in the time-domain multiscale problems.**

Keywords—the discontinuous Galerkin time-domain method; time-domain; multiscale problems; dispersive.

I. Introduction

At present, the discontinuous Galerkin time-domain (DGTD) [1-2] method is very popular to time-domain electromagnetic problems because of its high order accuracy, high parallel technology and flexible time iteration schemes. However, realistic electromagnetic wave propagation problems are often multiscale [3] due to complex geometries or heterogeneous media [4]. In fact, there are some locally refined meshes in large-scale multiscale problems. Since the maximal time step is limited by the smallest elements, traditional DGTD methods have the huge time consumption and memory usages. In order to avoid effectively these defects, we have carried out following research:

Considering there are huge numbers of meshes in large-scale multiscale problems and among them are coarse meshes, which is natural to use the explicit time scheme due to element's independence. However, there are still some refined meshes with small size resulting in stability constraint on explicit time schemes. In order to overcome the stability restrictions, we propose a time integration strategy by combining excellent stability properties with a new explicit time scheme. The specific ideas are as follows: start from the Lawson method, we develop a family of exponential-based time integration methods [5] that remove the stiffness on the time explicit integration of the semidiscrete operator associated with the fine part of the mesh, and allow for the use of high order time explicit scheme for the coarse part operator. The developed exponential time integration can be time advancing by a variety of explicit time stepping schemes; we adopt here a low-storage Runge-Kutta (LSRK) [6] scheme. Thus the so-called combined Lawson-LSRK time integration is constructed.

On the other hand, inhomogeneous media could bring multiscale problems due to taking into account the spatial discontinuity and non-uniformity of frequency and collision frequency. In order to solve effectively the dispersive problems with refined meshes, we introduce the Drude-type [7] based on the previous Lawson-LSRK, and develop a high order exponential-based time integration by DG schemes with central and upwind fluxes. As well as develop it to adapt Perfectly Matched Layer (PML) [8] for the open domain problems, this time integration has the almost without accuracy loss. As a result, this method remains the independence of the explicit time schemes and can give rise to a larger time step than existing explicit time schemes, which reduces greatly CPU time consumption.

However, it is inevitable to exist some refined meshes that have very small size during the large-scale multiscale problems. For such meshes, the implicit time scheme [9] has a greater advantage because of its unconditional stability compared with the explicit time scheme. But, the implicit time scheme needs to solve a global linear system. In particular, the coefficient matrix may be highly ill-conditioned, which makes it harder when dealing with high-order problems. Considering the hybridizable discontinuous Galerkin (HDG) frequency-domain method [10] has the fewer number of the globally coupled degrees of freedom (DOFs) than DG, we develop it into time domain based on the implicit time scheme, and form a global system that is only related to hybrid variables. Further, by applying the p-type multigrid preconditioner to accelerate the solution of global system, we can obtain a higher computational performance.

Finally, we propose a new strategy in the sight of meshes. For some large-scale multiscale problems with the complex microstructures and heterogeneous geometries, computational consumption is very huge[11]. Although there are some researches about non-conformal DGTD method [12], they are still preliminary. In order to reduce the cost, we propose the DGTD-MHM method by combining the DGTD with the multiscale hybrid method (MHM). MHM is a new kind of frame of multiscale domain decomposition method [13], which is based on a global problem on the boundary of macroscopic domain and some independent local problems on the interior domains. Once obtaining multiscale basis functions by solving the local problems, the global problem can be solved. Since the local problems are independent, hybrid mesh technique can be used. Besides, the DGTD-MHM is very suitable to implement parallel technology, and that's what we need to do in future.

The above methods have been applied different kinds of multiscale electromagnetic problems. Here we just give some numerical results about the scattering of a plane wave by an aircraft with 1430959 tetrahedral meshes. It is clear to see that the proposed DGTD- $\mathbb{P}_2$ method based on the combined Lawson-LSRK scheme is in good agreement with that of the fully explicit LSRK scheme in Figure 1, which further validates the numerical accuracy of this new explicit time scheme. And from Table I, the proposed method has the 10 times larger time step size and yields almost 10 times speed up. Besides, the peak memory usage of the proposed method is very low. Therefore, we can know that new strategies have the accurate and stable solution and a better computational performance compared with the traditional DGTD methods, which shows that recent developments to the DGTD are very promising for large-scale multiscale problems.

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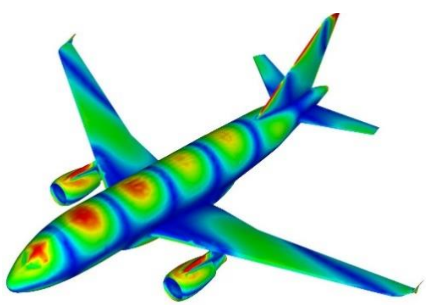
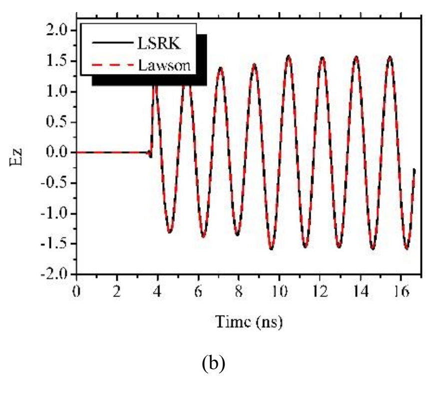


(a)



(b)

Fig. 1. Scattering of a plane wave by an aircraft at 600MHz obtained by the DGTD- $\mathbb{P}_2$ method based on the combined Lawson-LSRK scheme: (a) a top view about the contour lines of the Fourier transform of the electric field. (b) time evolution of E_z of a given point obtained.

TABLE I
SCATTERING OF A PLANE WAVE BY AN AIRCRAFT: PERFORMANCE FIGURES OF THE DGTD- $\mathbb{P}_k$ METHODS BASED ON THE COMBINED LAWSON-LSRK SCHEME VERSUS THE FULLY EXPLICIT LSRK SCHEME

$\mathbb{P}_k$	$\frac{\Delta t_{Lawson}}{\Delta t_{LSRK}}$	CPU (h)			Peak Mem (GB)	
		LSRK	Lawson	Gain	LSRK	Lawson
$\mathbb{P}_1$	10	27.7	2.9	9.6	13.4	14.0
$\mathbb{P}_2$	10	82.3	8.5	9.7	55.7	58.7

Note: Table I shows the performance statistics for 1 period by the DGTD- $\mathbb{P}_k$ methods based on the combined Lawson-LSRK scheme and the fully explicit scheme.

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