A Simple Method for Including the Antenna Pattern in Interfered Wireless Communications

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***Abstract*—This paper proposes a simple but accurate technique for modeling the antenna pattern in wireless communications. The technique relies on an analytical procedure based on a nonuniform discretization of the transmitting/receiving gains to reliably estimate the power received from an interfering source. The presented approach is validated by simulations considering terrestrial and space communication links.**

I. INTRODUCTION

In wireless networks, the modeling of the radiation pattern adopted by the communicating nodes represents a critical issue, since its oversimplification may lead to inaccurate results. A widely considered approximation is the flat-top model [1], which leads to manageable frameworks, but neglects most of the pattern features, such as the shape of the lobes and the depth of the nulls. The inclusion of these elements requires the usage of the actual pattern [2], at the cost, however, of an increased analytical complexity. For next-generation networks, the tradeoff between pattern modeling accuracy and complexity becomes relevant, since current proposals specifically focus on the antenna system to support inter-satellite [3], [4], and directional 5G links [5], [6]. These scenarios, in which multiple communications coexist, make fundamental the estimation of their reciprocal interference, whose evaluation cannot ignore the actual pattern shape.

To address this issue, this paper proposes a method for including the antenna pattern in wireless communications. The method, whose accuracy is validated by simulations, is applied to terrestrial and space scenarios, obtaining simple numerical expressions for the statistic of the received power while maintaining the antenna pattern exactly as it is.

The paper is organized as follows. Section II introduces the modeling approach. Section III presents the applications, while Section IV summarizes the main conclusions.

Notation: $\mathbb{1}_{\mathbf{X}}(x)$ denotes the indicator function (i.e., $\mathbb{1}_{\mathbf{X}}(x)=1$ if $x\in\mathbf{X}$, $\mathbb{1}_{\mathbf{X}}(x)=0$ if $x\notin\mathbf{X}$); $\gamma(\cdot,x)$ denotes the lower incomplete gamma function; $J_1(x)$ denotes first-order Bessel function of the first kind; for a random variable (rv), uppercase letter identifies the rv and lowercase its realization.

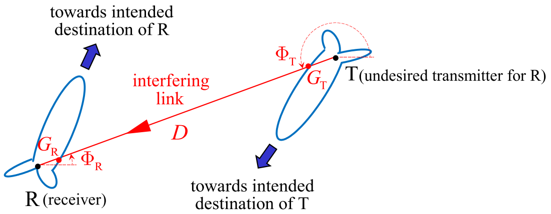

Fig. 1. Scenario.

II. MODELING APPROACH

Consider a scenario with randomly located nodes where two of them, T and R, communicate with the respective destinations, thus interfering with each other (Fig. 1). The directions of observation $\Phi_{\mathrm{T}}/\Phi_{\mathrm{R}}$ for R/T depend on the statistical node location, thus the transmitting/receiving gains $G_{\mathrm{T}} = G_{\mathrm{T}}(\Phi_{\mathrm{T}})$ and $G_{\mathrm{R}} = G_{\mathrm{R}}(\Phi_{\mathrm{R}})$ are rvs. These rvs determine the undesired received power [7]:

$$U = P_{\mathrm{T}} G_{\mathrm{T}} G_{\mathrm{R}} \left[\lambda/(4\pi D)\right]^{\alpha} \Psi = G \cdot (\beta\Psi/D^{\alpha}), \tag{1}$$

where P_{T} is the transmission power, λ is the wavelength, D is a rv with cumulative distribution function (cdf) $F_D(d)$ describing the $\mathrm{T}-\mathrm{R}$ distance, α is the path-loss exponent, Ψ is a rv with probability density function (pdf) $f_\Psi(\psi)$ modeling the power fluctuation (fading, shadowing, ...), $G = G_{\mathrm{T}}G_{\mathrm{R}}$ is the product gain, and $\beta = P_{\mathrm{T}}[\lambda/(4\pi)]^{\alpha}$. The cdf $F_P(p)$ of $P = \beta\Psi/D^{\alpha}$ can be calculated from $F_D(d)$ by first evaluating the cdf of $A = \beta/D^{\alpha}$ as:

$$F_A(a) = \Pr\{A \le a\} = 1 - F_D\left[(\beta/a)^{\frac{1}{\alpha}}\right], \tag{2}$$

and then using the product distribution, thus obtaining [2]:

$$F_P(p) = \int_0^{+\infty} F_A\left(p/\psi\right) f_\Psi(\psi)\mathrm{d}\psi. \tag{3}$$

The cdf in (3) may be analytically derived in many cases by exploiting several well-established expressions for $F_D(d)$ and $f_\Psi(\psi)$ [1], [2], thus, in (1), our objective is to focus on the estimation of the pdf $f_G(g)$ of G, so as to enable the derivation of the cdf of U without introducing simplified antenna models.

To this aim, consider an interval $[b_1, b_2]$, including all the possible values of $G_{\mathrm{T}}(\Phi_{\mathrm{T}})$ and $G_{\mathrm{R}}(\Phi_{\mathrm{R}})$, and subdivide it into L adjacent subintervals of equal length $\chi = (b_2 - b_1)/L$. Subsequently, calculate, for $l = 1, ..., L$ and $\mathrm{N} \in \{\mathrm{T}, \mathrm{R}\}$ (i.e., for both the transmitting and receiving patterns), the number of directions M_l^{N} in which $G_{\mathrm{N}}(\Phi_{\mathrm{N}})$ assumes values inside the subinterval $g_l = [10^{b_1+(l-1)\chi}, 10^{b_1+l\chi}]$, so as to estimate, for $\mathrm{N} \in \{\mathrm{T}, \mathrm{R}\}$, the pdf of G_{N} as:

$$f_{G_{\mathrm{N}}}(g) \cong f_{G_{\mathrm{N}}}(g_l) = \frac{M_l^{\mathrm{N}}}{\sum_{l=1}^{L} M_l^{\mathrm{N}}},\ l = 1, ..., L. \tag{4}$$

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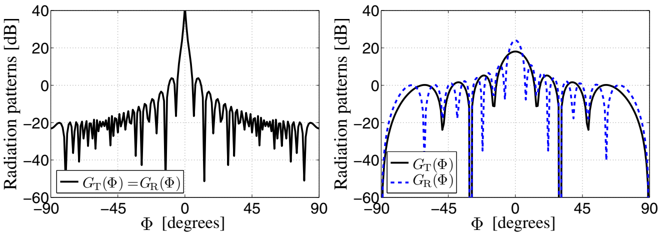
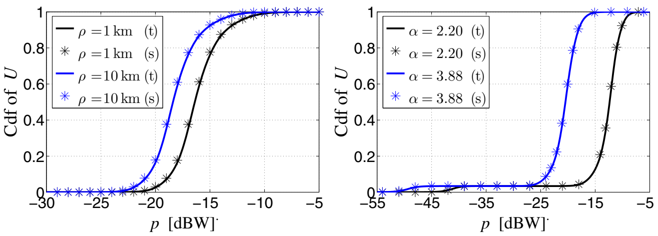


(a) (b)

Fig. 2. Modeled patterns: (a) example 1, (b) example 2.



(a) (b)

Fig. 3. Estimated cdfs (t: theory, s: simulation): (a) example 1, (b) example 2.

Observe that the actual discretization adopted for the gain values is nonuniform, in order to properly account for the shape of the main and secondary lobes as well as for the depth, width, and sharpness of the nulls. Thus, the normalization condition for the pdf in (4) is $\sum_{l=1}^{L} f_{G_\mathrm{N}}(g_l) g_l = 1$, and the pdf of the product gain $G = G_\mathrm{T} G_\mathrm{R}$ is given by:

$$f_G(g) \cong f_G(g_l) = \sum_{l'=1}^{L} f_{G_\mathrm{T}}(g_{l'})\, f_{G_\mathrm{R}}\!\left(\frac{g_l}{g_{l'}}\right),\ \ l = 1, ..., L. \quad (5)$$

In this way, the cdf of U can be estimated from the ratio distribution on a non-uniformly discretized domain as [2]:

$$F_U(u) \cong \sum_{l=1}^{L} F_P\left(u/g_l\right) f_G(g_l) g_l. \quad (6)$$

III. APPLICATIONS

The results are derived for $P_\mathrm{T} = 0.1$ W, $\lambda = 1$ cm, using an exponential pdf for the power variation (Rayleigh fading) [1]:

$$f_\Psi(\psi) = \exp(-\psi)\, \mathbb{1}_{[0,+\infty[}(\psi), \quad (7)$$

and adopting the distance distribution [8]:

$$F_D(d) = \left[2\,(d/\rho)^2 - (d/\rho)^4\right] \mathbb{1}_{[0,\rho]}(d) + \mathbb{1}_{]\rho,+\infty[}(d), \quad (8)$$

which derives from the random waypoint mobility model, a spatial statistic often adopted in network simulators to describe the node location inside a disk of radius ρ. By inserting (8) in (2), and then substituting the result in (3) together with (7), one obtains, after some manipulations, $F_P(p)$ in analytical form as:

$$F_P(p) = \left[\sum_{k=0}^{2} \frac{(-1)^k}{\rho^{2k}} \binom{2}{k} \left(\frac{\beta}{p}\right)^{\frac{2k}{\alpha}} \gamma\!\left(\frac{2k}{\alpha} + 1, \frac{\rho^\alpha p}{\beta}\right)\right] \mathbb{1}_{]0,+\infty[}(p). \quad (9)$$

Within this context, we first apply the pattern modeling method to an inter-satellite link with $\alpha = 2$ (example 1). The transmitting/receiving satellites employ the same parabolic reflector of efficiency $\varepsilon = 0.6$ and aperture $\Delta = 50$ (in wavelengths), whose pattern (Fig. 2(a)) is given by:

$$G_\mathrm{N}(\phi) = 4\varepsilon \left|\frac{J_1(\pi\Delta\sin\phi)}{\sin\phi}\right|^2 \mathbb{1}_{[-\frac{\pi}{2},\frac{\pi}{2}]}(\phi),\ \mathrm{N} \in \{\mathrm{T}, \mathrm{R}\}. \quad (10)$$

Fig. 3(a) reports $F_U(u)$ for $\rho = 1$ km and $\rho = 10$ km. The second application (example 2) considers a 5G uplink in a cell of radius $\rho = 100$ m. The nodes adopt two different uniform linear arrays of K_N isotropic $\lambda/2$-spaced elements lying on the x-axis, whose patterns (Fig. 2(b)) are given by:

$$G_\mathrm{N}(\phi) = \left|\frac{\sin(K_\mathrm{N}\pi\sin\phi/2)}{\sin(\pi\sin\phi/2)}\right|^2 \mathbb{1}_{[-\frac{\pi}{2},\frac{\pi}{2}]}(\phi), \quad (11)$$

where $K_\mathrm{T} = 8$ for the transmitting user and $K_\mathrm{R} = 16$ for the receiving base station. Fig. 3(b) shows the cdf of U for $\alpha = 2.20$ (line-of-sight conditions) and $\alpha = 3.88$ (non-line-of-sight conditions) [7]. All results are derived adopting $b_1 = -100$, $b_2 = 10$, $L = 1101$, and using Matlab on a Dell Latitude E5520 with an Intel Core i5-2520M @ 2.50 MHz. In Fig. 3(a,b), the significant matching between the theoretical curves (lines) and the simulated ones (markers) prove the accuracy of the proposed pattern modeling approach, which, furthermore, has required less than 4 s to provide the results.

IV. CONCLUSIONS

A simple but accurate method for including the transmitting/receiving antenna patterns in wireless communications has been proposed. The method, validated by simulations using different pattern shapes, has found to be suitable for quantifying the link performance in terrestrial and space scenarios.

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