BEM-FEBI Formulation Through 'PECfication'

Nash Lochner and Marinos N. Vouvakis
Department of Electrical and Computer Engineering
University of Massachusetts, Amherst, MA 01003, USA

***Abstract*—A multi-region formulation between Finite Element-Boundary Integral (FE-BI) regions and perfectly electric conducting (PEC) boundary element method (BEM) regions is presented. The method proceeds by first transforming the problem into a PEC equivalent one using a custom direct matrix solver within FE-BI regions, and then solving for the electric currents residing on FE-BI and BEM regions iteratively. The full field solutions within those 'PECfied' FE-BI regions can then be recovered using those electric currents and the matrix factors from the previous steps. Numerical results for two-dimensional radiation problems are presented.**

I. Introduction

The FE-BI method has been proven to be efficient in reducing the size of a computation domain when solving for unbounded electromagnetic problems through the enforcement of an exact radiation boundary condition [1], and allowing for coupling between FE-BI regions and BEM regions through the free space Green's function by placing equivalent electric and magnetic currents on the boundaries of FE-BI regions. Although the computational domain is reduced (virtually no free space need be discretized), the resulting FE-BI/BEM hybrid system of equations can be poorly conditioned; adversely affecting the performance of iterative solvers. Furthermore, due to the dense nature of BEM systems, fast and memory efficient methods must be considered which don't allow for a readily available factorization when considering direct methods.

To navigate the caveats of both the FE-BI and BEM methods, the outward looking radiation boundary conditions of [1] is considered, where the electric fields within FE regions are solved for by leveraging the memory efficient Direct Domain Decomposition Method (DDM) of [2], while the surface currents are found by 'PEC-ifying' the original system as seen in Figure 1, and solving the reduced system. In this work all the BEM regions will be considered to be purely PEC.

II. Theory

In general, an open computational domain my be split into K FE-BI regions and N BEM regions with a total of $M = K + N$ regions. Through enforcement of the outward looking radiation boundary conditions of [1], the resulting system takes the form

$$\begin{bmatrix} \boldsymbol{A} & \boldsymbol{D} \\ \boldsymbol{K} & \boldsymbol{L} \end{bmatrix} \begin{bmatrix} \boldsymbol{e} \\ \boldsymbol{j} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f} \\ \boldsymbol{b} \end{bmatrix}, \tag{1}$$

where $\mathbf{A}_{K\times K} = diag(\mathbf{A}_1, \mathbf{A}_2, \cdots \mathbf{A}_K)$ is the generalized system corresponding to the FE regions within all FE-BI regions. $\mathbf{D}_{K\times M}$ is the coupling between electric currents, $\mathbf{j}$, on all FE-BI and BEM-PEC surfaces with the electric fields, $\mathbf{e}$,

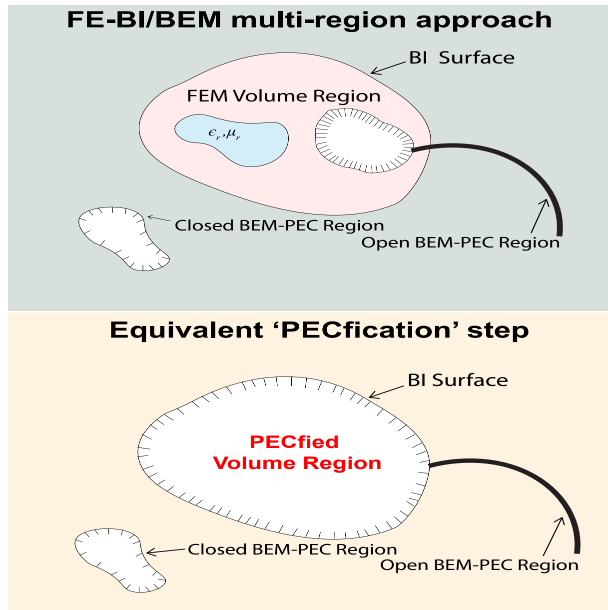


Fig. 1. A Generic multi-region FE-BI and BEM model and its reduction to a 'PEC-fied', i.e. PEC only, equivalent model.

on all FE-BI surfaces. $\mathbf{K}_{M\times K}$ is the Magnetic Field Integral Equation (MFIE) impedance matrix, coupling electric currents on all FE-BI and BEM-PEC surfaces with the electric fields on the surface of all FE-BI regions. Finally, $\mathbf{L}_{M\times M}$ is the Electric Field Integral Equation (EFIE) impedance matrix, coupling all electric currents on FE-BI and BEM-PEC surfaces. On the right hand side, $\mathbf{f}$ and $\mathbf{b}$ are the FEM and BEM-PEC system excitations, respectively. It is noted that the form of $\mathbf{D}$ in this abstract notation is different for interactions between the fields and currents residing on the same FE-BI surface and those residing on separate FE-BI and BEM-PEC surfaces.

In our approach, the system is first reduced to its PEC equivalent, through the Schur complement of Equation 1

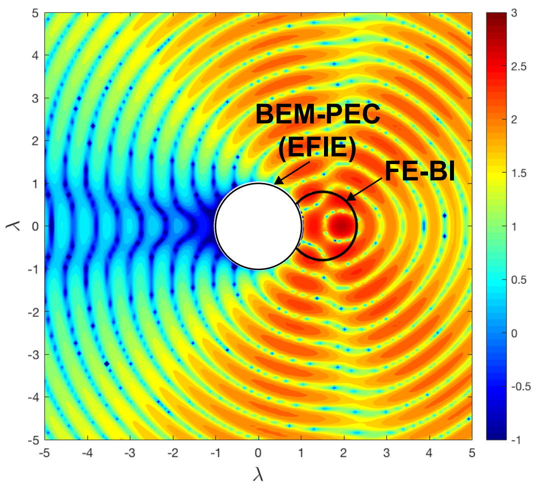
$$(\boldsymbol{L} - \boldsymbol{K}\boldsymbol{A}^{-1}\boldsymbol{D})\boldsymbol{j} = \boldsymbol{b} - \boldsymbol{K}\boldsymbol{A}^{-1}\boldsymbol{f}, \tag{2}$$

where we propose using a highly efficient DDM solver to first factorize $\mathbf{A}$, and then use forward-backward substitution to form the $\mathbf{K}\mathbf{A}^{-1}\mathbf{D}$ product. Equation 2 is then solved for iteratively to produce the electric currents, which can then be used to recover the electric fields within FE regions by once again solving for

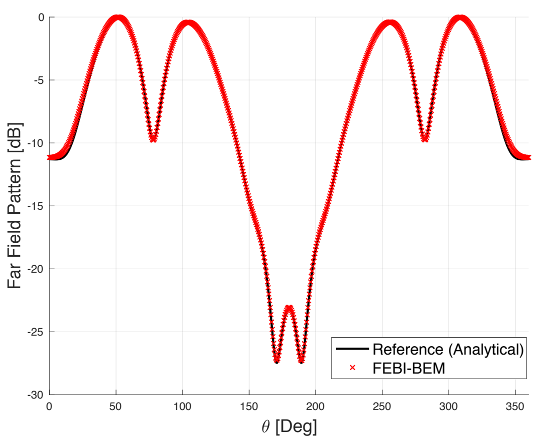
$$\boldsymbol{A}\boldsymbol{e} = \boldsymbol{f} - \boldsymbol{D}\boldsymbol{j}. \tag{3}$$

III. Numerical Results

To verify the accuracy and effectiveness of the proposed multi-region FE-BI/BEM approach, two two-dimensional problems were examined. The first has an analytical solution and is that of the radiation of an infinite electric line source in the presence of an infinite conducting circular cylinder, as shown in Figure 2(a). The same figure also shows the two regions where FE-BI and BEM-PEC is used and the real part of the electric field in log scale, showing the tangential continuity of the field distribution across the artificial FE-BI interface. Figure 2(b) compares the computed far-field with the analytical far-field showing almost perfect agreement. The discretization for this problem was kept to $h = \lambda/20$.



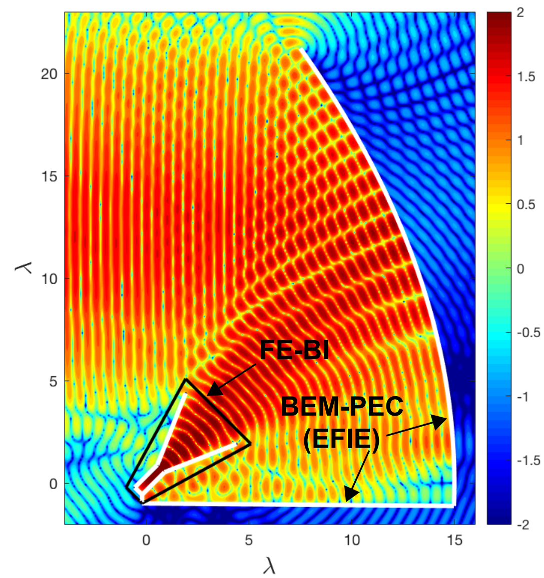
(a)



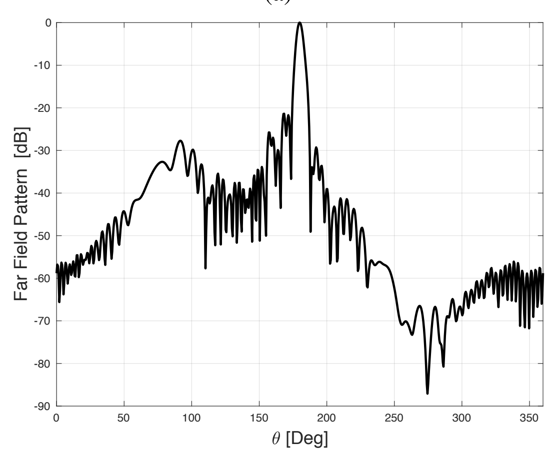
(b)

Fig. 2. Radiation (TM_z polarization) of an infinite electric line source in the proximity (1λ away) of a 2λ diameter infinite circular cylinder. (a) Geometry and magnitude of the real part of the E_z field, plotted in log scale. (b) Far-Field pattern of composite structure compared to the analytical solutions.

The second problem consists of a 20 λ offset parabolic reflector with mounting straps fed by a horn of 4λ aperture as shown in Figure 3(a). Again this is a two-dimensional problem for TM_z polarization, using nodal basis functions for both FE-BI and BI calculations. The near fields calculated outside of the FE-BI (indicated by a black border) region were computed by integrating the equivalent electric and magnetic surface currents placed on the boundary of the FE-BI region and the physical electric currents residing on BEM regions, while the fields shown within the FE-BI region are explicitly computed by FE-BI. Again the fields display tangential continuity across the FE-BI interface. Finally, the far-field of the offset parabolic reflector system is shown in Figure 3(b).



(a)



(b)

Fig. 3. Radiation (TM_z polarization) of an 2D horn/reflector system. (a) Geometry and magnitude of the real part of the E_z field, plotted in log scale. (b) Far-Field pattern of composite structure.

References

[1] A. F. Peterson, S. L. Ray, and R. Mittra, *Computational Methods for Electromagnetics*. New York, NY: Oxford, 1998.

[2] J. Moshfegh, M. N. Vouvakis, "Direct domain decomposition method (D3M) for finite element electromagnetic computations," *2016 IEEE International Symposium on Antennas and Propagation (APSURSI)*, 2016.

[3] J.-M. Jin, *Theory and Computation of Electromagnetic Fields, 2nd ed.*. John Wiley & Sons, 2015.