A Novel Dipole-Moment-Based Approach for Analyzing Scattering from Quasi-Periodic Structures

Kapil Sharma
MathWorks, Inc.
Natick, MA, USA
kapilsharma.psu@gmail.com

Raj Mittra
University of Central Florida
Orlando, FL, USA
rajmittra@ieee.org

***Abstract*—This work presents a novel approach, based on the dipole moment (DM) method, for analyzing electromagnetic scattering from quasi-periodic structures. The main advantage of using the DM method is that it provides convenient closed-form expressions for the scattered fields, facilitating the matrix computation much more efficiently than when the conventional MoM RWGs are used for this computation. Numerical results obtained by using the approach are compared to those derived from a commercial MoM software package, and good agreement is found.**

Keywords—Dipole Moment approach; quasi-periodic structure; Method of Moments (MoM); MoM matrix computation.

I. INTRODUCTION

In this work, we discuss a novel Dipole-Moment (DM) -based approach for numerically efficient solution of problems involving periodic structures. The DM approach provides us closed-form expressions for the scattered fields, and is far more efficient than the conventional RWG-based methods that utilize the Green's function technique instead for the field computation. It is well known that periodic structures are widely used in several applications such as electromagnetic bandgap (EBG) surfaces [1], frequency selective surfaces (FSSs) [2], phased arrays [3], to name a few applications [4]. Solution of problems involving periodic structures is computationally expensive when conventional Method of Moments (MoM) technique is utilized since the number of unknowns is often prohibitively large. To circumvent this problem, it is common to work with doubly-infinite periodic structures and utilize the periodic boundary condition (PBC) for the unit cell. Obviously this assumption ignores the truncation effects, and introduces inaccuracies in the numerical solution to the truncated periodic problem.

The proposed technique begins by representing each element of the quasi-periodic structure by an equivalent dipole moment, whose properties are found by matching the far fields scattered by the element when radiating in free space to those scattered by the corresponding dipole moment(s) in free space [5]. The key point is that the scattered fields from the equivalent DMs are available in closed forms, and the corresponding MoM matrix is computed for the quasi-periodic array in a very efficient manner. The scattered fields obtained by using the proposed DM approach have been compared with those obtained by using commercial MoM software (FEKO) to demonstrate the accuracy of the proposed approach.

II. DIPOLE MOMENT BASED FORMULATION

An arbitrarily shaped unit element of the quasi-periodic structure is represented by an equivalent dipole moment as shown in Fig. 1. The typical size of the element of a Metasurface (MTS) is on the order of λ/10, where λ is the free-space wavelength at the operating frequency.

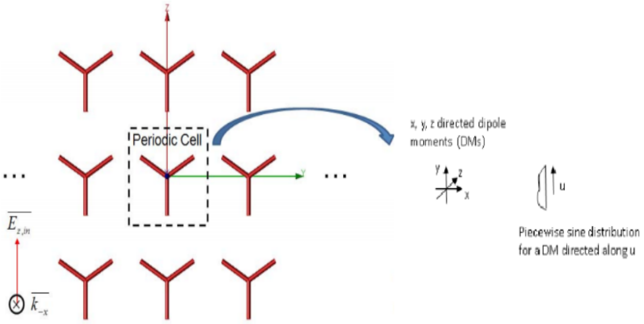

Fig. 1. Dipole moment formulation for quasi-periodic structures.

The induced current distribution in the unit-cell element is first computed by using a commercial MoM solver when it is operating either in free-space or an infinite periodic environment. Next, the far field radiation pattern for the element is computed from the above current distribution. Following this the unit-cell element is represented by x-, y-, and z-directed components of the dipole moments with piecewise-sine current distributions. The weight coefficients of the three DM components are computed next, by matching the far fields produced by the unit-cell element to those produced by the dipole moment representations of the same, as detailed in [5], by using (1) through (4).

$$w_1\overline{P_1} + w_2\overline{P_2} + w_3\overline{P_3} = \bar{P} \tag{1}$$

where w_1, w_2, and w_3 are the weight coefficients of x-, y-, and z-directed dipole moments with far field patterns $\overline{P_1}$, $\overline{P_2}$, and $\overline{P_3}$ respectively. The parallel component of scattered far fields from a $\bar{u}$-oriented dipole moment is given as:

$$E_u = -j30I_m\left(\frac{e^{-j\beta r}}{r} + \frac{e^{-j\beta r}}{r} - 2\cos(\beta_0 H)\frac{e^{-j\beta r}}{r}\right) \tag{2}$$

The expressions for other field components are also available in closed-form.

We can show that the patterns $\overline{P_1}$, $\overline{P_2}$, and $\overline{P_3}$ are orthogonal to each other, and this property is used to find the DM weight coefficients as follows:

$$w_1 = \frac{<\overline{P_1^*},\overline{P}>}{<\overline{P_1^*},\overline{P_1}>}, w_2 = \frac{<\overline{P_2^*},\overline{P}>}{<\overline{P_2^*},\overline{P_2}>}, w_3 = \frac{<\overline{P_3^*},\overline{P}>}{<\overline{P_3^*},\overline{P_3}>} \tag{3}$$

where the inner product of patterns $\overline{P_\alpha}$ and $\overline{P_\beta^*}$ is given as:

$$< \overline{P_\alpha}, \overline{P_\beta^*} >= \int_0^{2\pi} \int_0^{\pi} \overline{P_\alpha} * \overline{P_\beta^*}\, r^2\, sin\theta d\theta d\emptyset \tag{4}$$

Finally, the MoM matrix elements for the problem at hand, namely the truncated or quasi-periodic structure are computed by evaluating the dot product of the closed-form scattered fields from the source dipole moment(s) and the test dipole moment(s) as illustrated in Fig. 2.

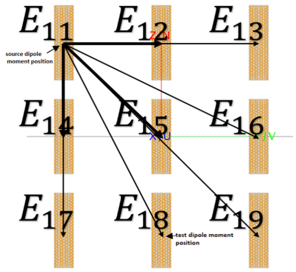

Fig. 2. Matrix calculation using the proposed dipole moment approach.

The coefficient vectors of the DMs are now derived from the solution of the MoM matrix, and are used to compute the fields scattered by the truncated structure.

III. NUMERICAL RESULTS

To demonstrate the accuracy of the proposed approach, the near fields of a unit-cell element comprising of a tilted circular loop are computed by using the proposed dipole moment approach, and the results are compared with those from a commercial MoM solver (FEKO). The geometry of the tilted loop problem is shown in Fig. 3.

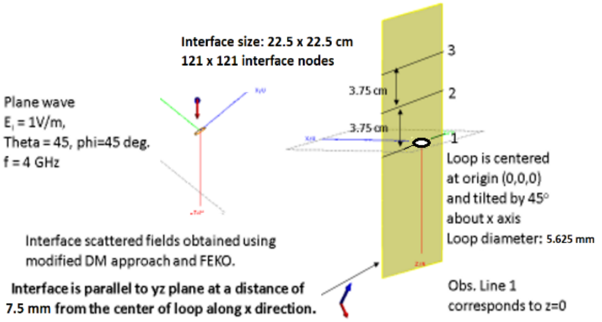

Fig. 3. Geometry of the tilted circular loop problem.

The comparison of the near fields along the observation line-1 is shown in Fig. 4. through Fig. 6. We note that the scattered field results along observation line-1, obtained by using the DM approach, agree well with the results derived from the FEKO solver.

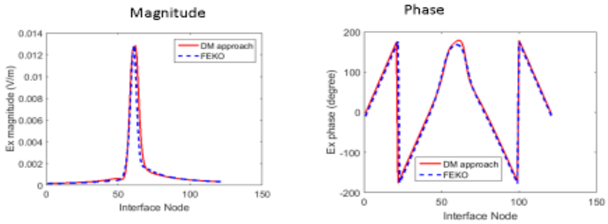

Fig. 4. Ex-field comparison for tilted circular loop along observation line-1.

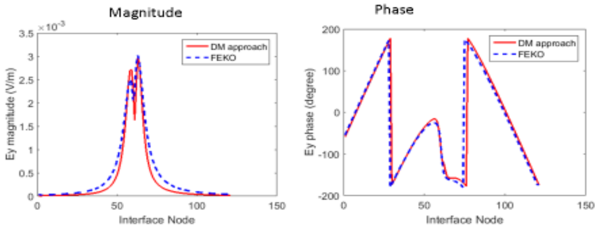

Fig. 5. Ey-field comparison for tilted circular loop along observation line-1.

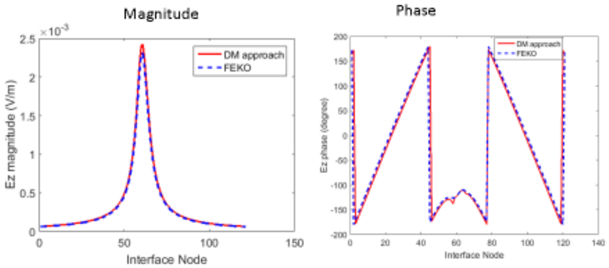

Fig. 6. Ez-field comparison for tilted circular loop along observation line-1.

Since the near field calculations agree so well, the final results for the scattered fields are also good as expected. The efficiency factor, defined as the ratio of the computational times of the MoM matrices generated by using the DM approach and a commercial MoM solver, is found to be on the order of 10^3 for a 10x10 truncated periodic structure problem, and it becomes even more favorable as we increase the screen's dimensions.

ACKNOWLEDGMENT

The authors are pleased to acknowledge collaboration with their colleagues at Loughborough University and Queen Mary, University of London, under the sponsorship of the SYMETA project.

REFERENCES

[1] L. Yang, M. Fan, F. Chen, J. She, and Z. Feng, "A novel compact electromagnetic-bandgap (EBG) structure and its applications for microwave circuits," IEEE Transactions on Microwave Theory and Techniques, vol. 53, no. 1, pp. 183 – 190, 2005.

[2] R. Mittra, C. H. Chan, and T. Cwik, "Techniques for analyzing frequency selective surfaces—A review," Proc. IEEE, vol. 76, no. 12, pp. 1593–1615, 1988.

[3] G. Yang, J. Li, R. Xu, Y. Ma, and Y. Qi, "Improving the Performance of Wide-Angle Scanning Array Antenna with a High-Impedance Periodic Structure," IEEE Antennas and Wireless Propagation Letters, vol. 15, pp. 1819-1822, 2016.

[4] R.-B. Hwang, Periodic Structures: Mode-Matching Approach and Applications in Electromagnetic Engineering. Wiley–IEEE Press, 2012.

[5] K. Sharma, "Techniques for Numerically Efficient Analysis of Multi-Scale Problems in Computational Electromagnetics," Ph.D. Dissertation, The Pennsylvania State University, 2017.