**Integral- and differential-equation magneto-quasi-static solvers for non-linear dynamics in magnetic materials and devices**

Simon Couture[1] and Vitaliy Lomakin[1]
(1) University of California, San Diego, CA 92093

Electromagnetic phenomena related to the magnetization dynamics is of interest for a range of applications, such integrated inductors, soft materials used in power applications, and magnetic write heads. For example, eddy currents generated by high speed magnetization dynamics lead to non-linear hysteresis losses, increased magnetic losses in high-permeability materials, increased line width of ferromagnetic resonances, and modified magnetization switching behavior. The ability to model such non-linear systems is important for our ability of their understanding and design.

The non-linear nature of the interactions often characterizing magnetic materials requires the need of coupling the Maxwell equations with the Landau-Lifshitz-Gilbert (LLG) equation, with the latter serving as a constitutive relation. Solving such a coupled system is a challenging task. The challenges are related to the need to overcome possible time domain instabilities arising from strong exchange fields characterizing ferromagnetic materials, possible inaccuracies and instabilities in the electromagnetic field dynamics, and the high computational cost of evaluating various numerical operators.

Here, we present and compare integral and differential equation based solvers for coupled magneto-quasi-static Maxwell's equations with the LLG equation to describe the magnetization dynamics accounting for electromagnetic interactions. In these formulations, the magnetic field component of the electromagnetic field is added as an additional effective field in the non-linear time-domain LLG equation. The time variation of the magnetization from the LLG equation serves as a source in the Maxwell equations. The two equations sets are solved at every time step simultaneously. The time solver is based on backward differential formulas with an implicit predictor-corrector framework. Newton iterations are used to address the magnetization non-linearities. Analytical formulas are derived to compute the system Jacobian and efficient preconditioners are used for accelerating the convergence of the linear solvers needed at each Newton step. The solver is time step and order adaptive to address the fact that the magnetization dynamics time scales may vary from nearly static to over 10 GHz. The Maxwell equation component is solved via either integral equation (IE) or differential equation. The integral equation is cast in three modifications, including a vector potential IE, electric field IE, and magnetic field IE. We find that the magnetic field IE is most robust but requires a slightly greater cost per time step. The spatial convolutions required for the IE is accelerated via the precorrected FFT methods, which is optimized for running on multi-core CPU and GPU computing architectures. The differential equation solver is formulated for the magneto-quasi-static correction of the Maxwell equations. All the formulations are implemented on multi-core CPU and on GPU computing architectures. The integral and differential equation solvers may be more or less efficient depending on the problem type. The IE formulations are typically of a higher accuracy and more efficient on GPU systems, which is attributed to a greater ability to parallelize spatial convolutions on GPU architectures.