On Parallel Direct Domain Decomposition Methods (D^3M)s

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Over the last decade, High Performance Computing (HPC) and parallel computing have been one the main drivers of computational electromagnetics methods (CEM). Yet, parallelization remains the key challenge for sparse direct solvers, arising in Partial Differential Equation (PDE) based solvers such as FEM. State-of-the-art sparse direct solvers such as MUMPS (P. R. Amestoy, et al, SIAM J. Matrix Anal. Appl., 23, 1, 2001) and PARDISO (O. Schenk, et al, Future Gener Comput Syst., 20, 3, 2004) don't scale favorably, although they employ elaborate multiple levels of parallelism and pipelining. Recently, research trends in dense direct solvers, e.g. PLASMA and DPLASMA, have shifted towrad directed acyclic graph scheduling to achieve efficient asynchronous parallel execution. However, a straight-forward adaptation of such approach to sparse matrices would lead to enormous number of highly irregular tasks, posing a major computational bottleneck. Hierarchical DAG scheduling has been introduced to overcome this challenge (K. Kim, V. Eijkhout, ACM Trans. on Math. Software, 41, 1, 2014), but it is only suitable for *hp*–FEM problems.

The memory efficient direct DDM (D^3M) (J. Moshfegh, and M. N. Vouvakis, IEEE APS, 2017) forms a small blocked sparse matrix of auxiliary unknowns defined on the domain interfaces. Then, uses a special block LDL^T with restricted pivoting for effient factorization. In this framework, computation of DtN map of domains can be done "embarrassingly parallel", whereas the special block LDL^T is suitable for asunchronous parallel execution, using a block directed acyclic graph (B-DAG) task scheduling. Using this parallelization method, one can achieve significant time saving. Results illustrating the validity and efficiency of the method on real-world scattering and radiation problems will be presented.