Pareto optimality of Q-factor and radiation efficiency in electrically small antennas

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Performance of electrically small antennas is often quantified by fractional bandwidth and radiation efficiency. These parameters are known to decay rapidly with the electrical size of the radiator. Here, we analyze physical bounds on Q-factor and radiation efficiency as well as the optimal trade-off between them (Gustafsson *et al.* 2017). The bounds are formulated as generalized eigenvalue problems using matrices derived from the method-of-moments impedance matrix. The trade-off between Q-factor and radiation efficiency is given by the Pareto front when a weight factor is introduced between the stored energy and ohmic losses. Solutions are presented for the cases with external tuning and self resonance. Moreover, it is demonstrated that the same procedure can be used to analyze important practical cases where only a small part of the radiator is used for the antenna design.

It is shown that the maximal radiation efficiency η for a given region comes with a high cost in Q-factor and is not self-resonant. Lowering the Q-factor is associated with a reduced radiation efficiency and the Pareto front spans a large range on both Q and η. The Pareto front under self-resonant constraint spans a much smaller range in these metrics and, considering electric surface currents only, there is generally almost no trade-off between Q and η . The minimum Q-factor is found to be self-resonant current in all but a few limiting cases (Capek *et al.* 2017), and by studying Pareto optimality it is observed that these currents have close-to-optimal radiation efficiency. Moreover, the Q-factor and radiation efficiency behave differently for volumetric electric current densities. Here, the minimum Q-factor does not change when the inner volumetric region is utilized , *i.e.*, it is sufficient to consider surface currents for minimum Q, however, contrary, the maximum radiation efficiency increases when volumetric currents are included. These differences increase the range of the Pareto front and produce a larger trade-off between Q-factor and radiation efficiency.

In this presentation, the aforementioned results will be presented along with the underlying formulation and used methodology . Practical implications and theoretical points of interest arising from the fundamental trade-offs between Q-factor and radiation efficiency will be highlighted to show how such calculations impact both applied design and fundamental understanding.