EM Modes for Model Order Reduction and Antenna Optimization

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***Abstract*—Expansions of fields and currents in modes are classical techniques for analysis of scatterers, antennas, and cavities. The modes often provide understanding of radiation properties and resonances. Here, we describe generalized modes constructed by iteratively solving an optimization problem with constraints of orthogonality to previous modes. This is used to construct modes that *e.g.*, have minimum Q-factor or maximal efficiency. The modes are illustrated for a planar rectangular shape and compared with characteristic and energy modes.**

I. Introduction

Modes are often used to expand electromagnetic fields and analyze scattering from spheres, propagation in waveguides, and cavity resonances. Characteristic modes have features such as orthogonal far fields and eigenvalues indicating resonances [1], [2], [3]. Although, these properties are useful for many problems [4] the orthogonal far field can be a disadvantage in problems involving near fields or the input impedance.

Here, a few alternative modes as well as a general procedure to construct modes are described. Energy and efficiency modes are constructed analogously to the characteristic modes by interchanging the reactance matrix to stored energy and ohmic loss matrices, respectively. These modes have orthogonal far fields similarly to characteristic modes, whereas the eigenvalues indicate normalized stored energy and ohmic losses, respectively. The orthogonal far fields cause the number of accurately determined modes to be low [5] and hence makes it difficult to accurately expand an arbitrary current density in modes. This cause negligible errors in the radiated field but the errors in the near fields and input impedance can be large. In these cases, it can be advantageous to replace the orthogonal far fields with orthogonal current densities. These modes are created iteratively by solving the original eigenvalue or optimization problem [6], [7], [8] for the lowest mode and removing the subspace associated with this mode. Here, we use this approach to construct a set of orthogonal current modes that are self-resonant and has deceasing efficiency. Numerical examples indicate that the modes can be useful to synthesize realistic antenna currents and for model order reduction [9].

II. Matrices and quadratic forms

A method-of-moments formulation of the electric field integral equation (EFIE) is used to determine the impedance matrix [10] $\mathbf{Z}=\mathbf{R}+\mathrm{j}\mathbf{X}=\mathbf{R}_{\mathrm{r}}+\mathbf{R}_{\Omega}+\mathrm{j}(\mathbf{X}_{\mathrm{m}}-\mathbf{X}_{\mathrm{e}})$. The quadratic forms for the stored magnetic and electric energies are [11], [12], [13]

$$W_{\mathrm{m}}\approx\frac{1}{8}\mathbf{I}^{\mathrm{H}}\left(\frac{\partial\mathbf{X}}{\partial\omega}+\frac{\mathbf{X}}{\omega}\right)\mathbf{I}=\frac{1}{4\omega}\mathbf{I}^{\mathrm{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I}\tag{1}$$

and

$$W_{\mathrm{e}}\approx\frac{1}{8}\mathbf{I}^{\mathrm{H}}\left(\frac{\partial\mathbf{X}}{\partial\omega}-\frac{\mathbf{X}}{\omega}\right)\mathbf{I}=\frac{1}{4\omega}\mathbf{I}^{\mathrm{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I},\tag{2}$$

respectively, where ω denotes the angular frequency and $\mathbf{I}$ the column matrix with expansion coefficients [10]. The Q-factor is determined from stored energies and dissipated power as

$$Q=\frac{2\omega\max\{W_{\mathrm{m}},W_{\mathrm{e}}\}}{P_{\mathrm{r}}+P_{\Omega}}\approx\eta\frac{\max\{\mathbf{I}^{\mathrm{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I},\mathbf{I}^{\mathrm{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I}\}}{\mathbf{I}^{\mathrm{H}}\mathbf{R}_{\mathrm{r}}\mathbf{I}},\tag{3}$$

where η denotes the radiation efficiency

$$\eta=\frac{P_{\mathrm{r}}}{P_{\mathrm{r}}+P_{\Omega}}=\frac{1}{1+\delta}\approx\frac{\mathbf{I}^{\mathrm{H}}\mathbf{R}_{\mathrm{r}}\mathbf{I}}{\mathbf{I}^{\mathrm{H}}(\mathbf{R}_{\mathrm{r}}+\mathbf{R}_{\Omega})\mathbf{I}},\tag{4}$$

P_{r} the radiated power and P_{Ω} the power dissipated from ohmic losses [14]. Resistive sheets with surface resistance $R_{\mathrm{s}}=1/(\sigma d)$, where σ is the conductivity and d the sheet thickness, are used to model the losses as $P_{\Omega}\approx\frac{1}{2}\mathbf{I}^{\mathrm{H}}\mathbf{R}_{\Omega}\mathbf{I}=\frac{R_{\mathrm{s}}}{2}\mathbf{I}^{\mathrm{H}}\mathbf{\Psi}\mathbf{I}$, where $\mathbf{\Psi}$ has the elements $[\Psi_{mn}]=\int_{\Omega}\boldsymbol{\psi}_m(\boldsymbol{r})\cdot\boldsymbol{\psi}_n(\boldsymbol{r})\,\mathrm{dS}$.

III. Mode expansions

Characteristic, energy, and efficiency modes are constructed from generalized eigenvalue problems [1] of the form

$$\mathbf{A}\mathbf{I}_n=\lambda_n\mathbf{R}_{\mathrm{r}}\mathbf{I}_n,\tag{5}$$

where we assume symmetry $\mathbf{A}=\mathbf{A}^{\mathrm{T}}$ and positive semi definiteness $\mathbf{R}_{\mathrm{r}}=\mathbf{R}_{\mathrm{r}}^{\mathrm{T}}\succeq\mathbf{0}$. Characteristic modes are determined using $\mathbf{A}=\mathbf{X}$, energy modes $\mathbf{A}=\mathbf{X}_{\mathrm{e}}+\mathbf{X}_{\mathrm{m}}$, and efficiency modes $\mathbf{A}=\mathbf{R}_{\Omega}$. The modes are orthogonal with respect to $\mathbf{A}$ and $\mathbf{R}$.

Optimal currents can be determined from optimization problems of *e.g.*, the form [15]

$$\begin{aligned}&\text{maximize} && \mathbf{I}^{\mathrm{H}}\mathbf{R}_{\mathrm{r}}\mathbf{I}\\&\text{subject to} && \mathbf{I}^{\mathrm{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I}=2\bar{P}_{\mathrm{w}}\\& && \mathbf{I}^{\mathrm{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I}=2\bar{P}_{\mathrm{w}}\\& && \mathbf{I}^{\mathrm{H}}\mathbf{R}_{\Omega}\mathbf{I}\leq2\bar{P}_{\Omega}.\end{aligned}\tag{6}$$

These problems are either convex or can be reformulated in convex form [15]. The solution of (6) gives the optimal value and a current that reaches this optima. We add a constraint $\mathbf{K}_p\mathbf{I}=\mathbf{0}$ to (6) such that the solution is orthogonal to previous modes. This new optimization problem is reduced to (6) by

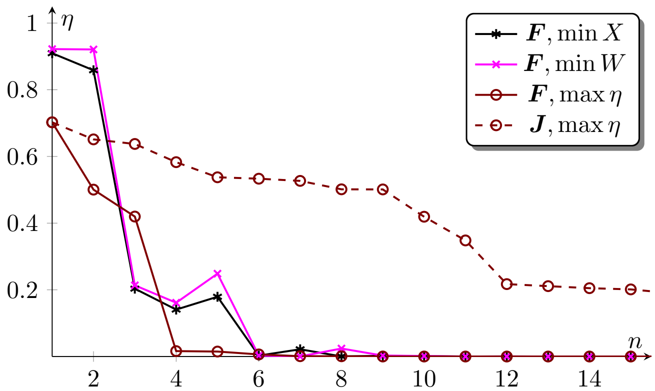

Fig. 1. Efficiencies for the first 15 modes on a planar rectangle for the characteristic ($\boldsymbol{F}$, $\min X$), energy ($\boldsymbol{F}$, $\min W$) and $\max \eta$ cases. The modes are orthogonal with respect to radiated far field $\boldsymbol{F}$ and current density $\boldsymbol{J}$.

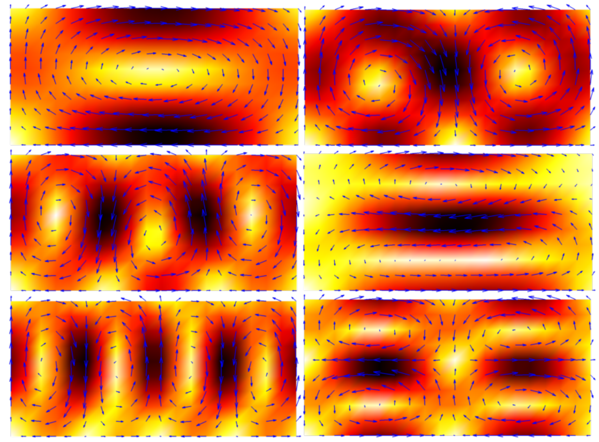

Fig. 2. Current density for the first 6 modes of the self-resonant maximum efficiency modes and orthogonal current densities ($\boldsymbol{J}$, $\max \eta$).

elimination of the affine constraint $\mathbf{K}_p\mathbf{I} = \mathbf{0}$ and can hence be solved analogously.

The dual of (6) can sometimes be written as a one dimensional optimization problems involving generalized eigenvalue problems

$$\lambda_n = \max_{\nu} \min_{\mathbf{I}_n^{\mathrm{H}}\mathbf{A}\mathbf{I}_m=\delta_{mn}} \frac{\mathbf{I}_n^{\mathrm{H}}\mathbf{X}_\nu\mathbf{I}_n}{\mathbf{I}_n^{\mathrm{H}}\mathbf{R}\mathbf{I}_n} \tag{7}$$

with $\mathbf{A} = \mathbf{R}$ and $\mathbf{A} = \mathbf{\Psi}$ for orthogonal far fields and current densities, respectively, where maximum self-resonant efficiency uses $\mathbf{X}_\nu = \nu\mathbf{X} + \mathbf{R}_\Omega$. This are easily solve by iteratively reducing the dimension of the current and matrices.

IV. Examples

Efficiencies for the first 15 modes are shown in Fig. 1 for a planar rectangle with side lengths ℓ and $\ell/2$ and electrical size $\ell = 0.2\lambda$. The characteristic and energy (5) modes have similar Q-factors are efficiencies for this case. The reduced efficiencies for the optimized modes are due to the enforced self-resonance compared with the tuned case for the characteristic and energy modes [8], [15]. The distribution of the modes are depicted in Fig. 2.

V. Conclusion

Generalized modes that combine optimality and orthogonality are presented. These modes are constructed by solving a standard optimization problem for some antenna quantity together with the affine constraint of orthogonality with respect to the previous modes. The modes can be used as an alternative to characteristic and energy modes for antennas problems and particularly for model order reduction.

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