Super-resolution in FDTD: Accurate Simulations with Coarse Grids

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In signal processing and imaging studies, the problem of super-resolution consists in the determination of high-frequency features of an image from low-frequency samples, collected (for example) by a low-pass instrument. Numerical methods such as the Finite-Difference Time-Domain (FDTD) similarly act as a low-pass measurement instrument, as numerical dispersion can significantly compromise the accuracy of their results beyond a certain frequency. Standard ways of dealing with the adverse effects of numerical dispersion range from mesh refinement to using high-order finite differences (N. Kantartzis, T.D. Tsiboukis, *Higher-Order FDTD Schemes for Waveguides and Antenna Structures*, Morgan-Claypool, 2006) and to applying signal processing methods (B. Donderici and F.L. Teixeira, *IEEE Trans. Antennas and Propagat.*, vol. 53, no. 9).

We present an alternative route to obtaining accurate results with FDTD at coarse grids, exploiting the mathematical theory of super-resolution, originally presented in (E. J. Candés, C. Fernandez-Granda, *Comm. Pure Appl. Math*, 67: 906-956). The main result in this theory is that a pulse train sampled in the frequency domain up to frequency f_c can be exactly recovered, as long as the pulses have a minimum separation distance of $2/f_c$. We discuss the implications of this condition and how it can be met in practical electromagnetic problems. Subsequently, we present a new approach to decompose an arbitrary FDTD output signal that does not meet the minimum separation condition into a sum of signals that individually do meet the aforementioned minimum separation condition and hence, can be reconstructed by solving a convex optimization problem.