Accelerating the Transmission Matrix Retrieval via Low-Rank Compression and Fast Proximal Gradient Methods

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The transmission characteristics of reciprocal linear devices and media are oftentimes characterized by their $\mathbf{S}_{12} = \mathbf{S}_{21}^T$ transmission matrices, which typically are determined using repeated measurements of modal amplitudes of transmitted (output) fields for different incident (input) excitations. Unfortunately, in many applications the direct measurement of transmitted wave amplitudes is impractical as the output aperture is not readily accessible. Under such conditions, only backscattered waves, generated by the media potentially in conjunction with scatterers residing beyond the output aperture, can be measured.

We recently proposed a novel approach to retrieve $\mathbf{S}_{21}$ of a random medium with the assistance of “guide stars,” viz. strong scatterers that reside beyond the medium’s output aperture (H. Guo et al., Radio Science Meeting USNC-URSI, 2017); similar techniques are often used in astronomy to calibrate telescopes. In a nutshell, guide stars generate backscattered fields containing information that enables the reconstruction of $\mathbf{S}_{21}$ from backscatter measurements. Specifically, we posed the $\mathbf{S}_{21}$ retrieval problem as a matrix equation $\mathbf{S}_{21}^T \cdot \mathbf{S}_g \cdot \mathbf{S}_{21} \cdot \mathbf{X} = \mathbf{Y}$, where $\mathbf{X}$ refers to the incident wave, $\mathbf{Y}$ models the backscattered wave, and $\mathbf{S}_g$ is the guide star’s backscattering matrix. We next proposed an efficient method leveraging proximal gradient techniques for solving this equation using $O(n^4)$ CPU resources assuming there are $O(n)$ guide stars. The method was much faster than classical methods for solving the above equation, which require $O(n^6)$ CPU resources, but remains hopelessly slow for applications involving apertures that can accommodate hundreds or even thousands of modes.

Here, we propose a two-prong approach to further accelerate the process of retrieving scattering matrices. First, we apply compression to the variable $\boldsymbol{\mathcal{X}} = \tilde{\mathbf{S}}_{21}^T \otimes \tilde{\mathbf{S}}_{21}^T$, exploiting the low-rank nature of the estimated transmission matrix $\tilde{\mathbf{S}}_{21}^T$ in each iteration of the proximal gradient method; 2) we introduce Nesterov's fast gradient method (FGM) (Y. Nesterov. Soviet Mathematics Doklady, 27 (2), pp. 372-376, 1983) as well as other recently proposed optimized gradient methods (OGMs) (D. Kim and J. A. Fessler, arXiv preprint arXiv:1608.03861, 2016) for speeding up the convergence of the proximal gradient method. Our numerical simulations indicate these enhancements yield significant improvements in $\mathbf{S}_{21}$ retrieval time over previous ones. We believe that the proposed techniques will favorably compete with existing biomedical techniques for retrieving $\mathbf{S}_{21}$ that require implantation of probes or inject radiating sources (e.g. fluorescent microspheres) into the medium. Indeed, such methods are invasive and inconvenient for many *in vivo* applications. The proposed technique does not suffer from any of these drawbacks.