Joint Electromagnetic and Acoustic Image Reconstruction using a Compressive Sensing Unmixing Algorithm

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Many energy-driven applications in earth science require the use of sensing and imaging technology to detect and monitor fluid flow, transport, and chemical reactions. The accuracy of this technology determines how efficiently and reliably one can extract hydrocarbons and coordinate the disposal of waste materials. Most systems utilize acoustic or seismic modalities in order to image underground processes, although electromagnetic sensing modalities have also been employed.

In this paper, we describe a novel approach for jointly reconstructing petro-physical media from electromagnetic and acoustic measurements. Our method considers the coupling between the two imaging modalities by formulating the inverse problem in an unmixing framework, wherein the content of each voxel is modeled as a combination of several material types (e.g. water, hydrocarbons, soil, etc.). By modeling the electromagnetic and acoustic properties of these material types, the imaging algorithm seeks to reconstruct the mixture proportions of each material at each voxel. When combined with novel compressive sensing (CS) imaging techniques, our approach can achieve enhanced reconstruction performance.

Formally, the reconstruction process can be expressed as the following optimization program:

$$\begin{aligned} \underset{\boldsymbol{z}_1,\ldots,\boldsymbol{z}_R}{\text{minimize}} \quad & \sum_{r=1}^{R} \|\boldsymbol{z}_r - \mathbf{v}_r\|_{\ell_1} \\ \text{subject to} \quad & \|\hat{\mathbf{y}} - \sum_{r=1}^{R} \mathbf{A}_r \boldsymbol{z}_r\|_{\ell_2} \leq \eta \\ & \boldsymbol{z}_r \succeq \mathbf{0}_N,\ r = 1,\ldots,R \\ & \sum_{r=1}^{R} \boldsymbol{z}_r = \mathbf{1}_N \end{aligned} \tag{1}$$

where $\boldsymbol{z}_1,\ldots,\boldsymbol{z}_R \in \mathbb{R}^N$ are the unknown mixture proportions for the R materials at each voxel, $\boldsymbol{v}_1,\ldots,\boldsymbol{v}_R \in \mathbb{R}^N$ are estimates of the mixture proportions obtained from a previous survey, $\hat{\mathbf{y}} \in \mathbb{C}^M$ is the measurement vector, and $\mathbf{A}_1,\ldots,\mathbf{A}_R \in \mathbb{C}^{M\times N}$ are the measurement matrices for each material type, which are computed using the Born approximation.