Some Investigations into the Dispersion Characteristics of Localized Bessel Beams

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An investigation into possible increases of maximal propagation distance of Bessel Beam will be presented. The Bessel Beams presented in this work will be produced by a leaky radial waveguide consisting of a capacitive sheet over a ground plane to create a normal electric field given by a truncated Bessel function(M. Ettorre and A. Grbic, *IEEE Trans. Antennas Propag.*, vol. 60, pp. 3605-3613, Aug. 2012). The required properties of leaky waveguides which best support non-diffracting waves are determined by the nth-order Hankel function of the first and second order, assuming TM modes. A significant feature of the localized nature of Bessel beams is the *dispersion* distance

$$\mathrm{Z}_{max} = \mathrm{a}\sqrt{\left(\frac{k_2}{\beta_\rho}\right)^2 - 1}. \tag{1}$$

Beyond Z_{max}, the localized nature of the Bessel beams start to degrade. A primary objective is to seek methods by which the dispersion distance can be enhanced. In (1) the β_ρ is the phase factor of the radial propagation constant $k_\rho = \beta_\rho - \jmath\alpha_\rho$, and, a is the radius of the circular aperture. The aperture radiates into medium # 2 defined by its intrinsic wavenumber k_2. Obviously, it follows from (1), that if β_ρ is decreased, and the circular aperture radius a is increased, then the dispersion distance can be enhanced.

To that end, by invoking the conditions of producing the pure Bessel ($\mathrm{J}_\circ(z)$) beam in the circular aperture, use is made of the following Hankel large argument forms, that read:

$$\mathrm{H}_\circ^{(1),(2)}(z) \approx \sqrt{\frac{2}{\pi z}}[\mathcal{P}(0,z) \pm \jmath\mathcal{Q}(0,z)]e^{\pm\jmath(z-\frac{\pi}{4})}. \tag{2}$$

In (2), $\mathcal{P}(0,z) \approx 1$, and $\mathcal{Q}(0,z) \approx \frac{-1}{8z}$. The argument $z = k_\rho\rho$ in (2). Invoking the conditions of producing the pure Bessel beam one then obtains the dispersion relationship

$$\tan(z - \frac{\pi}{4}) = 8z. \tag{3}$$

It is thus obvious that solutions to (3) shall lead to real values of β_ρ. This approach is different from the earlier work of Ettore and Grbic, *op. cit.* There could be several forms of the dispersion relationships that are distinct from (3), using different approximations to the Hankel functions, and hence a host of solutions for the radial propagation constant are feasible. These aspects shall be discussed in the view of the results from full-wave EM simulations for characterizing the dispersion distance of localized Bessel beams.