Parallel Sparse LU factorization for Domain Decomposition Global Preconditioning

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The global communication throughout the decomposed domains is the crucial computational component that guarantees the efficiency and robustness of the Domain Decomposition Methods. Woodbury Finite Element Tearing and Interconecting (W-FETI) (W.Wang, PhD thesis, University of Massachusetts Amherst, 2016) provides an elegant way of integrating local with global preconditioners.The global preconditioning component requires the solution of a sparse matrix system that is blockwise sparse and unsymmetric. For very large FEM models this part dominates the computation, nessecitating a good parallel direct solver.

The state-of-the-art parallel direct solvers for the unsymmetric sparse linear systems rely on the elimination or assembly tree that based on the symmetrized matrix layout of $A^T + A$, while they use multi-frontal (P. R. Amestoy , A fully asynchronous multifrontal solver using distributed dynamic scheduling, 2001) or the left/right looking super-nodal (O. Schenk and K. Gartner, Solving unsymmetric sparse systems of linear equations with PARDISO, 2004) algorithms to achieve data and temporal locality. Their parallel implementations use various mapping strategies such as subcube-to-subtree or proportional mapping schemes applied on the elimination tree in order to reduce the communication volume and ensure load balancing. Mapping phase followed by a static and/or dynamic scheduling algorithms. Besides the eminent parallel efficiency, their main drawbacks are the high critical path length (upper bound of the parallel performance) of the elimination tree and the poor exploitance of the multilevel parallelism (tree and node based parallelism).

The goal of this work is the implementation of a robust shared and/or distributed memory parallel sparse LU(right and/or left looking) factorization that can overcome the weaknesses of the state-of-the-art parallel direct solvers, leveraging the block-wise sparse layout of the W-FETI preconditioner. Considering each block of the W-FETI as different parallel task, a Directed Acyclic Graph(DAG) can be used to describe the task dependencies. Computation and communication cost models of the factorization and communication primitive operations can be used to determine the weights of the DAG facilitating the usage of an exact static scheduling heuristic algorithm(list scheduling or clustering) to guide our parallelization. Results showcasing the efficiency of the method on scattering and radiation of electrically large will be presented.