

Canonical Problems for Multifrequency Modulated Metasurfaces

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Artificial impedance surfaces, which can be realized by printed metasurface (MTS) technology, constitute a cost effective versatile solution to implement a number of devices [C. L. Holloway, et al., An overview of the theory and applications of metasurfaces: the two-dimensional equivalents of metamaterials, *IEEE Antennas Propagat. Mag.* vol. 54, no. 2, pp. 10-35, 2012.] for applications at microwave regime, particularly when strict constraints on devices overall profile and bulkiness apply. MTSs can be efficiently modeled by an equivalent boundary condition (BC) that can be of a scalar or anisotropic nature, linking the tangential components of the electric and magnetic fields.

An effective tailoring of the equivalent BC, translating into MTS features engineering, allows one to finely control the plane wave reflection or the propagation properties of a surface wave (SW). Also, such surface can become a customizable and effective mean to radiate power in the direction transverse to the surface by transforming a SW into a leaky wave (LW) when it is periodically modulated. Indeed, the problem of a SW propagating on a modulated impedance boundary can be considered as a canonical problem, whose solution assumes a great significance for MTS antenna design.

Within a famous seminal work [A. Oliner, A. Hessel, A. Guided waves on sinusoidally-modulated reactance surfaces. *IRE Transactions on Antennas and Propagation*, 1959, 7.5: 201-208.], Oliner and Hessel firstly addressed the canonical problem of an isotropic sinusoidally modulated sheet of impedance, meaning that the MTS is modeled by a scalar BC. By a recursive approach, the authors proposed a rigorous and analytical solution to such problem, allowing for determination of the dominant mode propagation wavenumber and of the equivalent leakage constant.

For more advanced MTS antenna applications, a fundamental requirement is constituted by the acquisition of the radiating mode polarization control for addressing improved pattern purity; this issue can be managed by switching to sinusoidally modulated anisotropic MTSs. In this case, and under analogous hypotheses to the isotropic case, the problem of an impinging SW can be generalized and modeled as an anisotropic 2D canonical problem where the equivalent sheet impedance assumes a tensorial form. The solution to this generalized canonical problem has been proposed in [F. Caminita, E. Martini, S. Maci, Rigorous Analysis of Sinusoidally-modulated Anisotropic Reactance Surfaces: Theory and Application to the Design of Modulated Metasurface Antennas, *EuCAP 2015*]. Without losing generality, the impedance can represent an impenetrable or a penetrable sheet posed within a dielectric stratification. More recently, as the interest for MTS antennas has grown, additional features were put under the scope, as for instance multi-frequency operability. For addressing this feature, more complex modulations, composed by two or more harmonics, should be considered. The resulting function, provided the fulfillment of some non-restrictive conditions, is still periodic and the supported field can be represented on a Floquet base.

The purpose of this paper is to propose a further canonical problem generalization for multi-harmonic anisotropic modulations and its relevant solution. This appears to be fundamental for study and development of multi-frequency MTS antennas, in particular for ensuring polarization purity and aperture efficiency similarly to more classical narrowband solutions.