Higher Order (in Time) Stable PWTD-Accelerated Time Domain Integral Equation Solver

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Time domain integral equation have seen dramatic advances in terms of both computational cost, memory, as well as stability. More specifically, while classical methods scale as $\mathcal{O}(N_t N_s^2)$ where N_t and N_s are the number of temporal and spatial degrees of freedom. Methods to ameliorate this cost have been developed, and are the plane wave time domain method (PWTD) (A. A. Ergin, B. Shanker, and E Michielssen, Journal of Computational Physics 146 (1), 157-180, 1998) and the time domain adaptive integral method (TDAIM) (A. E. Yilmaz, J. M. Jin, and E. Michielssen, IEEE Transactions on Antennas and Propagation, 52, 2692-2708, 2004). These methods reduced the cost to $\mathcal{O}(N_t N_s \log^2 N_s)$, and $\mathcal{O}(N_t N_c \log^2 N_c)$, where N_c is the number of auxiliary in grid points on the Cartesian grid. The second bottleneck is late time stability. Papers attempting to address this issue have been ongoing since the 1960's. It is only recently that some resolution has been brought to this issue; see (A. J. Pray *et. al.*, IEEE Transactions on Antennas and Propagation, 62, 6183-6191, 2014), (Chen *et. al.*, Comm. Comput. Phys. 11, 383-399, 2012) and reference therein. However, integration of fast methods with late time stable techniques is still a challenge, and this is elaborated next.

The challenge does no lie in the mathematics, but overcoming different approximations. Stable time domain methods use different approximations to effect interaction between basis functions. In Pray *et. al.*, the convolution of the space-time basis function with the retarded potential is approximated using a set of polynomials that is smooth and continuous over the domain of the triangle. Where as in PWTD, currents are mapped onto approximate prolate spheroidal wave functions, and then the retardation is effected via a plane wave expansion. A mapping on to a planewave expansion results in another temporal derivative. As a result of these two different approximations, we have found that integrating stable TDIE with PWTD to be unstable. In this work, we intend resolving this conundrum and will hopefully result in transient methods that are provably stable. The results of this investigation will be presented at the conference.