New FFT Subspace-based Optimization Methods for Inverse-Scattering Problems

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***Abstract*—This paper presents a new fast Fourier transform subspace-based optimization method (NFFT-SOM) for solving inverse-scattering problems. After the deterministic part of induced current is extracted out, a space spanned by the whole Fourier bases is used in the proposed NFFT-SOM during optimization, instead of adopting the subspace spanned by smaller singular vectors as in the traditional subspace-based optimization method (SOM). Compared with previously proposed SOM and its variants, the proposed NFFT-SOM presents three advantages: its overhead computational cost is low, it is more robust in presence of high noise, and it is much easier to implement.**

***Index Terms*— inverse scattering, microwave imaging, optimization.**

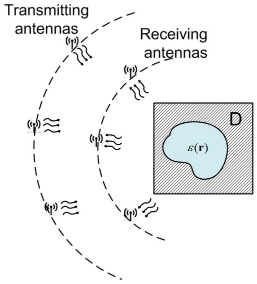


Fig. 1. The geometrical model for the inverse-scattering problems

I. INTRODUCTION

This paper addresses inverse-scattering problems, i.e., reconstructing permittivities of scatterers from scattered fields. Inverse-scattering techniques have wide applications in quantitatively determining either physical or geometrical properties in various fields. Inversion methods act as a powerful tool in non-destructive detections/testings. It is very challenging to solve full-wave inverse scattering problems due to their ill-posedness and nonlinearity. Several iterative inversion methods have been proposed to deal with them. For example, the distorted Born iterative method [1] and contrast source inversion method (CSI) [2] have been widely used.

Recently, an algorithm, termed subspace-based optimization method (SOM) [3], is proposed to deal with inverse scattering problems. In SOM, induced current is decomposed into deterministic part and ambiguous part. The former is obtained analytically and the latter is reconstructed within a subspace via optimization. It has been shown that SOM is very fast and robust against noise. However, since SOM requires a full SVD, the overhead computation cost is high. To reduce the computational cost and further improve the speed of SOM, some modified methods, such as improved subspace-based optimization method [4] and twofold subspace-based optimization method (TSOM) [5], have also been proposed.

In this paper, we propose a new fast Fourier transform subspace-based optimization method (NFFT-SOM) to solve inverse-scattering problems. The proposed method is a generalization of the inversion method for electrostatic electrical impedance (EIT) problems [6] to inverse-scattering problems. Compared with SOM, the proposed NFFT-SOM reconstructs the ambiguous part of induced current from complete Fourier bases instead of the subspace spanned by singular vectors corresponding to small singular values. Consequently, the NFFT-SOM has the following three advantages: low overhead computation, robust in present of high noise, and easy to implement.

II. NEW FAST FOURIER TRANSFORM SUBSPACE-BASED OPTIMIZATION METHOD

We consider a transverse-magnetic (TM) two-dimensional inverse-scattering problems, as shown in Fig. 1. Nonmagnetic scatterers are located in a domain of interest (DOI) D and illuminated by N_i line sources. For each incidence, scattered field is measured by N_s antennas located in far field. The DOI is discretized into M subunits. Upon using the method of moments (MOM), the state equation and data equation are obtained, respectively,

$$\overline{J} = \overline{\overline{\xi}} \cdot (\overline{E}^i + \overline{\overline{G}}_D \cdot \overline{J}) \tag{1}$$

$$\overline{E}^s = \overline{\overline{G}}_s \cdot \overline{J} \tag{2}$$

where $\overline{E}^i$, $\overline{E}^s$, and $\overline{J}$ are incident field, scattered field, and induced current, respectively. The diagonal matrix $\overline{\overline{\xi}}$ is the only parameter directly related to relative permittivities with $\overline{\overline{\xi}}_m = (\varepsilon_{rm} - 1)$ for the mth unit.

For inverse problem, in the traditional SOM, a full SVD is firstly conducted on $\overline{\overline{G}}_s$, i.e., $\overline{\overline{G}}_s = \sum_m \overline{u}_m \sigma_m \overline{v}_m^*$ with $\overline{\overline{G}}_s \cdot \overline{v}_m = \sigma_m \overline{u}_m$, $\sigma_1 \geq \sigma_2 \ldots \geq \sigma_M \geq 0$. The induced current $\overline{J}$ is mathematically classified into deterministic current $\overline{J}^s$ and ambiguous current $\overline{J}^n$, where the former is uniquely determined by the first L singular vectors and the latter is reconstructed in the subspace spanned by the remaining $M-L$ singular value vectors. However, as mentioned in [4], the

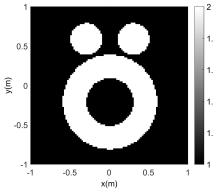


Fig. 2. Inverse scattering experiment of the Austria profile: exact profile. The grey bar shows the value of relative permittivity.

drawback of the traditional SOM is its overhead computational cost associated with a full SVD of $\overline{\overline{G}}_s$.

In this paper, we propose a new fast Fourier transform subspace-based optimization method that avoids a full SVD of $\overline{\overline{G}}_s$. At the same time, fast Fourier transform can be used to accelerate the computational speed. In NFFT-SOM, the deterministic current is still computed by the first L singular vectors, whereas the ambiguous current is spanned by a complete Fourier bases $\overline{\overline{F}}$. Since only the first L singular vectors are needed, a thin-SVD of $\overline{\overline{G}}_s$ suffices. The complexity of a thin SVD $O(MN_sL)$ is much smaller than that of a full SVD $O(M^2N_s)$. Thus, the induced current can be written in the form

$$\overline{J}_p = \overline{J}_p^s + \overline{\overline{F}} \cdot \overline{\alpha}_p^n \tag{3}$$

where $\overline{\alpha}_p^n$ is an M-dimensional vector to be reconstructed. With (3), the residual of (2) is

$$\Delta_p^f = ||\overline{\overline{G}}_s \cdot \overline{J}_p^s + \overline{\overline{G}}_s \cdot \overline{\overline{F}} \cdot \overline{\alpha}_p^n - \overline{E}_p^s||^2 \tag{4}$$

and residual of (1) becomes

$$\Delta_p^s = ||\overline{\overline{A}} \cdot \overline{\alpha}_p^n - \overline{B}_p||^2 \tag{5}$$

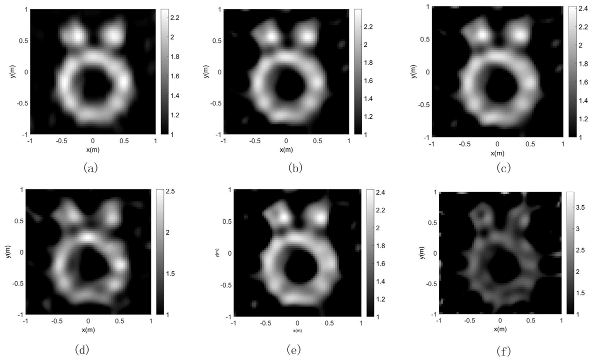
in which $\overline{\overline{A}} = \overline{\overline{F}} - \overline{\overline{\xi}} \cdot (\overline{\overline{G}}_D \cdot \overline{\overline{F}})$, and $\overline{\overline{B}}_p = \overline{\overline{\xi}} \cdot (\overline{E}^i + \overline{\overline{G}}_D \cdot \overline{J}_p^s) - \overline{J}_p^s$. The objective function is defined as

$$f_0(\overline{\alpha}_1^n, \overline{\alpha}_2^n, ..., \overline{\alpha}_{N_i}^n, \overline{\overline{\xi}}) = \sum_{p=1}^{N_i} (\Delta_p^f / |\overline{V}_p|^2 + \Delta_p^s / |\overline{J}_p^s|^2) \tag{6}$$

In this paper, conjugate gradient method is used to minimize the objective function. Note that the number of unknowns is equal to $(N_i + 1)M$, which is slightly more than that of the traditional SOM $N_i(M - L) + M$.

III. NUMERICAL RESULTS

The numerical example with the widely-used "Austria ring" is used to test the performances of the NFFT-SOM. The distribution of relative permittivity is shown in Fig. 2, the details of which can be found in [3]. For as high as 30% additive white Gaussian noise, the reconstruction results obtained by the NFFT-SOM and the traditional SOM at the 300th iteration for different choices of L are given in Fig. 3. It is obvious that NFFT-SOM outperforms the traditional SOM for L = 25. We also notice that NFFT-SOM seems to obtain visually better reconstruction results than the traditional SOM does for both L = 5 and L = 15.



(a) (b) (c)

(d) (e) (f)

Fig. 3. Reconstruction results for the NFFT-SOM (upper row) and the SOM (lower row) at the 300th iteration for different choices of L: L = 5 for (a) and (d); L = 15 for (b) and (e); L = 25 for (c) and (f).

IV. CONCLUSION

The proposed NFFT-SOM mainly has the following advantages. First, compared with SOM, the computational complexity of the proposed method is largely reduced since it avoids the full singular-value decomposition and in addition FFT can be directly used in algorithm; Second, The reconstruction results are more robust in presence of high noise since it is able to rectify the error generated in the deterministic part of induced current; Third, compared with TSOM, the proposed NFFT-SOM is much easier to implement. Specifically, only one projection is needed in NFFT-SOM rather than two as done in TSOM and in addition 1D Fourier bases are used in NFFT-SOM instead of 2D Fourier bases, which avoids extending the DOI if it is not a rectangle one.

ACKNOWLEDGMENT

This research was supported by the National Research Foundation, Prime Minister's Office, Singapore under its Competitive Research Program (CRP Award No. NRF-CRP15-2015-03).

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