First-Principles Statistical Model of Communication Through Wave-Chaotic Environments

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***Abstract*—We propose a rigorous and versatile mathematical model in prediction and exploration of wave propagation through complex wave-chaotic environments. The work investigates an innovative theoretical solution to Maxwell's Equations in the wave-chaotic media. The fundamental solution, named stochastic Green's function, rigorously integrates the coherent and incoherent propagations within a compact form. Furthermore, by incorporating the component-specific information with the universal chaotic dynamics, the work accomplishes an elegant and systematic framework for the statistical analysis and over-the-air testing of wireless communication. The theoretical investigation is evaluated and validated through representative experiments.**

I. MOTIVATION

Transmission of information through chaotic, random environments is a topic of both fundamental and practical importance to wireless communication. Representative applications include indoor communications within large and complicated enclosures, communications in metropolitan areas, radar systems over rough or glistening surfaces, transmission through diffusive random media, etc. As the wavelength is much shorter than the typical size of the structures in the environment, the wave scattering process may exhibit chaotic ray dynamics, even though the scattering environment is deterministic. On the other hand, the complexity of the scattering environment creates an abundance of multiple paths between transmitters and receivers. Attributed to the distinguished characteristics and unique behaviors, physicists and engineers have found diverse applications, including multiple-input and multiple-output (MIMO) communications, time-reversal systems, wavefront shaping and focusing, sensing and targeting, et al. These intriguing systems and experiments are realized by taking advantage of chaotic sensitivity, ergodicity, and broadband spectra of dynamics in phase space. Yet, much potential is not fully exploited due to a lack of rigorous and versatile mathematical theories and models.

II. METHODOLOGY

Evidently, the difficulty of determining precise environmental information and the high variability of wave properties lead to a conceptual limitation for deterministic approaches. This paper investigates a physics-oriented statistical wave model of communication through complex wave-chaotic environments. The methodology is to first establish fundamental statistical representations of complex wave-chaotic media, then integrate component-specific features of wireless antennas, and finally encode the governing physics into the mathematical information theory. The **original contributions** are as follows.

1) A novel stochastic Green's function (SGF) method is proposed, which quantitatively describes the universal statistical property of chaotic media through random matrix theory (RMT) [1]. Comparing to existing statistical electromagnetics approaches [2]–[7], the proposed work rigorously separates the coherent and incoherent propagations, and seamlessly incorporates universal statistical properties and deterministic coupling characteristics within a comprehensive form, which is first time available in the literature.

2) Based on the SGF, we develop a new stochastic integral equation (SIE) formulation that statistically replicates the chaotic interactions within wave-chaotic environments. It rigorously resolves the transmitting correlation, propagation correlation, and receiving correlation in the dense multipath environment. Moreover, it leads to nonstandard statistical models which utilize the macroscopic properties of environments instead of detailed environmental specifics.

3) We have derived a stochastic fast multipole representation of the SIE formulation. The coherent (specular direct paths) propagations between the source and receiver are represented by the deterministic translation operator. The effects of incoherent (diffuse multipath) chaotic rays with random phases are characterized by the random matrix theory.

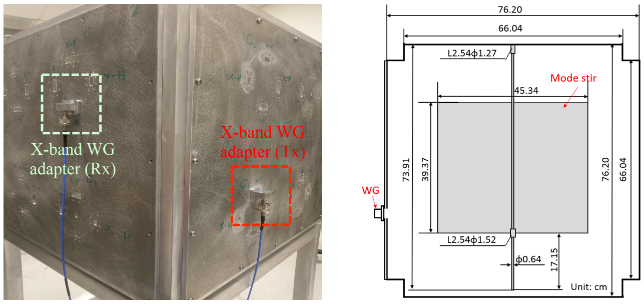
Finally, we propose a hybrid formulation to include the specific knowledge of the wireless antennas. The volumetric domain of antennas is discretized by the finite element method. The SIE-SGF is placed on the exterior surface of the antennas, which statistically emulates the propagation phenomena in the chaotic environment. The resulting ergodic information capacity for M^{T} transmitting (Tx) and M^{R} receiving (Rx) antennas can be represented as: $C = \log_2 \det\left(\mathbf{I}_{\mathrm{M}^{\mathrm{R}}} + \frac{\rho}{\mathrm{M}^{\mathrm{T}}}\mathbf{H}^{+}\mathbf{H}\right)$, where $\mathbf{I}_{\mathrm{M}^{\mathrm{R}}}$ is the Identity matrix and ρ is the average received signal-to-noise ratio (SNR). The $\mathbf{H}$ is the random transfer matrix from the Tx antennas to the Rx antennas, calculated from the proposed work. We remark that the $\mathbf{H}$ naturally encodes the deterministic features of the antennas, and statistical behavior of the propagating environment.

III. EXPERIMENTAL VALIDATION AND VERIFICATION

We first validate the proposed work with a confined EM system, a complicated 3D aluminum cavity. The geometry and photograph of the experimental setup are illustrated in

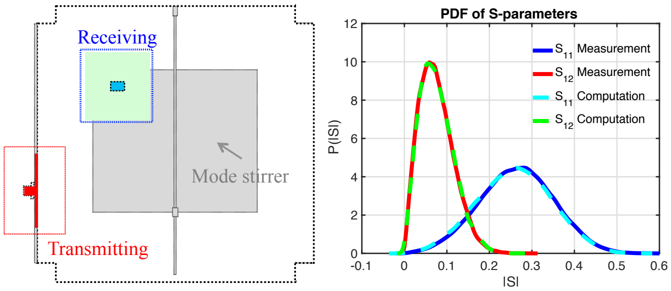
Fig. 1(a)-(b). Two X-band waveguide (WG) antennas are mounted on the adjacent walls as Tx and Rx. A paddle-wheel mode stirrer is used to generate a configuration ensemble of measurements. The cavity is significantly overmoded and EM fields exhibit wave chaotic fluctuations.

In the computational setup, we only need to include the site-specific features, namely, the WG antennas and their surrounding aluminum plates, as illustrated in Fig. 1(c). The interaction with other parts of the 3D cavity is categorized statistically with the SIE-SGF defined on the exterior surface of the antenna. As a results, the solution of the stochastic model can be obtained at the same cost as if the antenna radiating in free space. The probability distribution function (PDF) of the antenna S-parameters is depicted in Fig. 1(d), where an excellent agreement is observed.



(a) Experimental setup

(b) Configuration



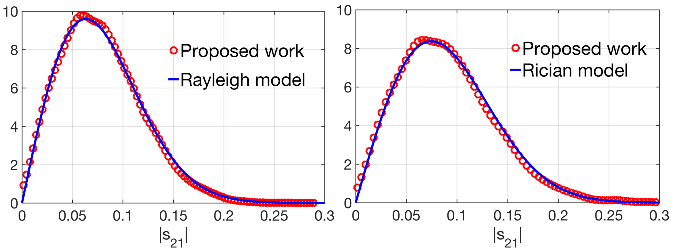
(c) Computational setup

(d) Probability density function

Fig. 1: Experimental validation of the stochastic wave model

We proceed to verify the work with semi-empirical statistical models, the Rayleigh fading and the Rician fading, which are introduced for non-line of sight (NLOS) and line of sight (LOS) cases in extremely multipath environments [8]. In the experimental verification, we consider two monopole antennas as Tx and Rx, separating at two different distances, (a) 20λ and (b) 0.75λ. The PDF of the normalized S-parameter is depicted in Fig. 2. In the well-separated case (a), the SGF is dominated by the incoherent component, which corresponds to the diffuse multipath propagator. The PDF indeed follows the Rayleigh NLOS distribution. In the case (b) of two nearby antennas, there are mixed specular direct path and diffuse multipath. The PDF agrees with the Rician LOS distribution.

Next, we calculate the PDF of the channel capacity with respect to different Signal-to-Noise ratios in Fig. 3. We notice that the results are consistent with the expression given in the literature for the single-input single-output case (SISO) in presence of Rayleigh fading [9]. Nevertheless, there are no fitting parameters needed in the proposed work.



(a) separation distance: 20λ

(b) separation distance: 0.75λ

Fig. 2: PDF of the normalized S_{12} parameter

(a) SNR: $\rho = 2$

(b) SNR: $\rho = 4$

Fig. 3: PDF of the information capacity

IV. CONCLUSION

We have proposed a novel first-principles statistical model for predicting the performance of wireless antennas in the multipath chaotic environment. It doesn't make the ansatz for special cases (isotropic fading, far-field separations, etc.), and there are no semi-empirical fitting parameters required. The advancements will result in a reliable, reconfigurable, and repeatable testbed for emerging wireless devices and systems in complex environments, beyond the confines of the laboratory and measurements.

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