

# Numerical Simulation of Cascaded Planar Metasurfaces Exhibiting Bianisotropic Properties

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Metasurfaces (MTSs) are electrically-thin metamaterial layers designed to exhibit unusual reflective/refractive properties or guided wave characteristics. The goal of this work is to develop a full-wave fast-solver (Method of Moments based on Fast Fourier Transform) for the design of metasurfaces with bianisotropic responses, with a special emphasis on modulated metasurfaces, i.e. aperiodic cell arrangements. Bianisotropic metasurfaces can be realized by cascading multiple isotropic/anisotropic electric sheet admittances (patterned metallic claddings), with dielectric spacers separating them. Here, we present a numerical, spatial domain approach to computing the electromagnetic response of cascaded planar electric sheet admittances by means of Surface Integral Equations (SIEs). Each sheet admittance ( $i$ ) is modeled as a zero-thickness electric discontinuity characterized by the following Impedance Boundary Condition (IBC):

$$\mathbf{e}^{av} = \mathbf{Z}_e^i \cdot \mathbf{j}^{av} \quad (1)$$

where  $\mathbf{Z}_e^i$  is a  $2 \times 2$  tensor, which can be anisotropic, and varying in space. The terms  $\mathbf{j}^{av}$  and  $\mathbf{e}^{av}$  represent the electric surface currents and the average electric field tangential to the surface  $\Sigma_i$ , respectively. Assuming an arbitrary incident field,  $\mathbf{e}^{inc}$ , we can write the Electric Field Integral Equation (EFIE) for planar cascaded metasurfaces as:

$$\hat{\mathbf{n}} \times (\mathbf{Z}_e \cdot \mathbf{j}) + \hat{\mathbf{n}} \times Z_0 \mathcal{L}(\mathbf{j}) = \hat{\mathbf{n}} \times \mathbf{e}^{inc} \quad (2)$$

where  $\mathcal{L}$  is the Electric Field Integral Operator (EFIO). A conventional Galerkin testing procedure is assumed. The overall domain  $\Sigma = \bigcup \Sigma_i$  is approximated by a mesh of planar triangles. The solution  $\mathbf{j}$  is approximated as  $\mathbf{j} \approx \sum_n j_n \mathbf{f}_n$  where  $\mathbf{f}_n$ ,  $n = 1, \dots, N$  are Rao-Wilton-Glisson (RWG) basis functions defined on the  $N$  internal edges of the mesh. The unit vector  $\hat{\mathbf{n}}$  is normal to the surface  $\Sigma$ .  $\mathbf{Z}_e$  is a matrix which accounts for the contribution of each impedance tensor  $\mathbf{Z}_e^i$ . It should be noted that the tensor  $\mathbf{Z}_e^i$  does not include the dielectric spacers. The spatial dispersion resulting from the electrical thickness of the cascaded metasurface (multiple electric sheet admittances separated by spacers) is taken into account by the Green's functions for multi-layered media. It is worth noting that cascaded metasurfaces can also be analytically modeled as a single bianisotropic Generalized Sheet Transition Condition (GSTC) with spatially dispersive surface parameters. In the design of bianisotropic metasurfaces, one typically goes from the single bianisotropic GSTC sheet to a realizable, cascaded metasurface consisting of multiple electric sheet admittances. This process can be performed analytically using Wave Matrices for periodic metasurfaces (A. Ranjbar and A. Grbic, Phys. Rev. B, 95, 205114, 2017), but becomes rather challenging for aperiodic designs. The goal of the proposed full-wave solver is to aid in this process, and avoid approximate strategies such as employing the local periodicity approximation.