# On a Low-Frequency and Refinement Stable Exact Kernel Wire-EFIE Formulation

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The Electric Field Integral Equation (EFIE) allows for efficient and accurate modeling of electric scattering by Perfectly Electrically Conducting (PEC) bodies. Being a Boundary Element Method based formulation, it only requires discretization of the scatterer boundaries, yielding smaller interaction matrices than its Finite Element Method counterparts. In the case of wire-like structures, assuming the absence of radial currents, this efficiency can be further improved by employing the Wire-EFIE, which allows modelization of wires with linear one-dimensional elements. In order to fully exploit these advantages hybrid surface-wire formulations have been widely implemented in commercial solvers. It should be noted that the Wire-EFIE can be derived from two different kernels: (i) the well-posed exact kernel and (ii) the reduced kernel which is computationally lighter but imposes additional constraints on the type of excitation and on the discretization of the geometry, we will focus on the former.

Despite its advantages the Wire-EFIE, being an Electric Field Integral Operator (EFIO)-based formulation, suffers from both a low-frequency and a high-refinement breakdown. Both breakdowns cause high conditioning of the interaction matrices and, as a consequence, slow down or prevent usage of iterative solvers. The low-frequency breakdown also has the side-effect of causing numerical cancellations in the solution of the equation, when computed in finite precision. It has traditionally been addressed by the so-called *Loop-Star* technique which leverages a quasi-Helmholtz (qH) decomposition of the loop and star parts of the discretized space to independently re-scale and store the loop and star parts of the equation. While it does address the low-frequency issues of the formulation, this technique is prohibitively expensive because it requires an at-best quadratic global loop search algorithm to be employed and it further degrades the high-refinement conditioning of the equation. The simulation slowdown caused by the intrinsic high-refinement ill-conditioning of the EFIO, has been compensated by a large family of variably invasive techniques, including wavelet-based matrix sparsification. To the best of our knowledge, no exact kernel Wire-EFIE formulation immune form both breakdowns has yet been presented.

We propose a new fully stable exact kernel Wire-EFIE formulation which relies on an extensive analysis of the spectral properties of the EFIO. The low-frequency breakdown is addressed by adapting to one-dimensional problems the recently introduced qH projectors that allow the same re-scaling as traditional Loop-Star without any of its drawbacks. A carefully chosen hierarchical preconditioner is then applied to the projected equation in order to regularize its high-refinement behaviour. The correctness and resilience of our formulation to both breakdowns have been conclusively verified with numerical simulations.