$\mathcal{H}$-Matrix Arithmetic for a New Single-Source Integral Equation for 3D Scattering from Homogeneous Objects

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In our previous work (A. Menshov, et.al., IEEE T-MTT, no. 1, vol. 61, pp. 341–350, 2013) and (F. Sheikh Hosseini Lori, et.al., in print in IEEE T-APS, 2017), the Surface-Volume-Surface Electric Field Integral Equation (SVS-EFIE) has been proposed for scattering problems on homogeneous dielectric objects. Compared to traditional surface integral-equation formulations, the SVS-EFIE reduces the number of unknowns by half; moreover, unlike the previously known single-source integral equations, the new equation requires only one product of integral operators and uses only electric-field-type of Greens function. However, the requirements on memory and high computational complexity of dense matrix operations and storage for naïve MoM solution of the SVS-EFIE limit its application to small problems only. In this work, we investigate a fast direct and iterative solutions for the SVS-EFIE based on the hierarchical matrix ($\mathcal{H}$-matrix) framework (W. Hackbusch, Computing, 62, 89–108, 1999) for scattering and radiation problems on homogeneous dielectric objects.

As the SVS-EFIE features the product of the integral operators mapping the tangential equivalent electric current on the surface of the scatterer to the volume polarization current and the integral operator mapping the volume polarization current to the tangential component of the scattered electric field, its MoM discretization produces the product of the non-square matrices. The $\mathcal{H}$-Matrix arithmetics used to form and operate with the rectangular matrices produced by the MoM discretization of the SVS-EFIE will be presented, as well as the associated computational and memory complexity estimates. The new computational framework allows for the iterative solution of 3-D scattering problems with $O(P^{1.5} \log P)$ CPU time and memory complexity and their direct solution with $O(P^{1.5} \log^2 P)$ CPU time and $O(P^{1.5} \log P)$ memory complexity, P being the number of surface unknowns produced by the MoM discretization. The proposed accelerated method is shown to provide an efficient compression of the MoM impedance matrix which leads to significant reduction of the memory usage and the CPU time. The method is particularly effective for the solution of radiation problems in bioelectromagnetics which require computation of the fields throughout the volume of the objects.

The algorithms for arithmetics pertinent to the product of the non-square $\mathcal{H}$-matrices as well as the memory and CPU time complexity scaling of the required operation will be presented. Finally, the optimized method will be applied to accelerate solution of scattering problems on biological objects found in the bioelectromagnetics applications.