Simplifying and Generalizing Antenna Array Expressions with the Antenna Equation

Everett G. Farr
Farr Fields, LC, Albuquerque, NM 87123

Our antenna definitions standard has almost no complex terms that include both magnitude and phase. Most terms, such as gain, are scalar. This potentially poses a problem for describing antenna phased arrays, which require phase information. One exception is effective length, which is the ratio the received open circuit voltage to the incident electric field (IEEE, *IEEE Standard Definitions of Terms for Antennas*, IEEE Std 145™–2013, Institute for Electrical and Electronics Engineering, New York, December 2013). Effective length does include phase information, and it is commonly used to describe antenna performance in those applications, such as phased arrays, where phase is required.

However, effective length is not completely general, and it produces equations that are unnecessarily complicated. Effective length is undefined for waveguide feeds, because one cannot measure an open circuit voltage. One can avoid the problem by adding waveguide-to-coax adapters, but the adapter response is then mixed in with the antenna response.

We simplify and generalize antenna array expressions with the newly developed antenna equation (E.G. Farr, "Characterizing Antennas in the Time and Frequency Domains," to be published in *IEEE Antennas and Propagation Magazine*, February 2018) and (E. G. Farr, "A Power Wave Theory of Antennas," *Forum for Electromagnetic Methods and Application Technologies* (online), Vol. 7, 2015, www.e-fermat.org). The antenna equation describes antenna performance in a manner that is both compact and elegant. It works in both the time and frequency domains, and in both transmission and reception. It also adds a meaningful phase to antenna gain. By adapting this to antenna arrays we achieve a significant simplification of the expressions used to describe antenna arrays, and we generalize the results to waveguide feeds.

An added benefit of using the antenna equation is that it shows how to generalize mutual impedance in an antenna array. Mutual impedance in an antenna array is the ratio of the open-circuit voltage produced at one terminal to the current supplied to a second terminal, when all other terminal pairs are open-circuited. This is undefined for waveguide feeds, because one cannot measure an open-circuit voltage. As before, one can add waveguide-to-coax adapters to all the ports, but the adapter response is then included with the antenna response. The antenna equation allows one to isolate the mutual coupling response from the adapter, using simpler equations than those used previously. This results in a new concept, a unique and well-defined mutual coupling coefficient in antenna arrays.