A Hybrid Integration Scheme to Handle Singularities in the MFIE

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For the rigorous solution of radiation and scattering problems using integral equation formulations, it is of crucial importance to develop numerical codes able to accurately model and predict the electromagnetic behavior of these problems. An accurate and efficient implementation of the method of moments (MoM) requires the accurate and efficient numerical evaluation of singular and near-singular potential integrals present in MoM system matrices.

One of the most common schemes to evaluate strongly near-singular integrals has been the so-called "singularity subtraction" approach. The idea is to subtract from the integrand terms having the same asymptotic behavior as the integrand at the singularities; then the difference integrand is integrated numerically by standard quadrature rules, and the subtracted singular terms are integrated analytically (see e.g. R. D. Graglia, IEEE Transactions on Antennas and Propagation, vol. 41, no. 10, 1993). The analytical calculation of the asymptotic behavior at the singularity provides good efficiency (fewer integration points) for a medium-low accuracy range. However, the calculation of the numerical integral limits the maximum accuracy that can be reached.

Another approach is the so-called "singularity cancellation" scheme, where instead of subtracting asymptotic singular terms from the integrand, the singularities are cancelled via appropriate variable transformations. Since the quadrature is performed numerically, a singularity cancellation scheme is essentially independent of bases, element shapes and curvature; moreover, essentially arbitrary accuracy is achievable by suitably increasing the quadrature order (see e.g. P. W. Fink et al., IEEE Antennas Wireless Propag. Lett.,. 7, 2008). This allows one to reach machine precision at the expense of lower efficiency (greater number of integration points) in the low-to-medium accuracy range.

This paper explores the possibility of combining these two-existing well-known schemes to evaluate strongly near-singular integrals with a $1/R^2$ singularity present, such as in the MoM discretization of the Magnetic Field Integral Equation (MFIE). First, a subtraction of the singularity of higher order is applied. The resulting integrand presents a singular term $1/R$, which is canceled via an appropriate variable transformation. The aim is to achieve good efficiency in the low-to-medium accuracy range and yet be able to reach machine precision by merely increasing the quadrature order. Preliminary results have demonstrated the possibility of achieving both goals. The method is relatively simple to implement, and it also appears the approach can be extended to curved elements.