The Least Common Multiple Sin-Cos Method for FDTD Simulation of Stratified Media at oblique Incidences with Different Frequencies

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The problem of plane wave incident on stratified media is one of the most important topics in electromagnetics. In the case of simple media, there are analytic solutions to the problem, but numerical methods such as FDTD are required when solving problems about complex materials such as time-varying or nonlinear materials.

In solving a problem using FDTD method, periodic boundary condition need to be applied quite often. Various types of methods for periodic boundary condition have been proposed so far. Among them, sin-cos method has been widely used because it is easy to implement and it is easy to reuse the existing (legacy) codes. The method generates two identical computational domains and uses them to calculate phase delay between periodic boundaries. It is known that the method is only applicable to solve the problem for single frequency. If we want to involve nonlinear phenomena between plane waves with different frequencies, however, the algorithm should be modified so that it can support two or more frequencies.

In this presentation, we propose the least common multiple (LCM) sin-cos method to analyze the problem where two plane waves having different frequencies enter at an arbitrary angle. The proposed method can be applied more effectively when one of the plane waves is normally incident. The theoretical interpretation is based on the Floquet mode theory. The main idea of proposed method is that periodicity can be chosen arbitrarily in stratified media. When we choose P as a periodicity in x-direction, the relationship of the field ψ(E or H) on both sides of the periodic structure is as shown in (1)

$$\psi(x=P,y,z)=\psi(x=0,y,z)\exp(-jk_x P) \qquad (1)$$

where, $k_x=k_{x0}+2\pi m/P$ (m=0,1,2,...). k_x is wave number of Floquet modes in x-direction and k_{x0} is wave number for the fundamental Floquet mode. Phase delay between periodic boundaries is $k_xP = k_{x0}P+2\pi m$. Second term of right side can be ignored because it is integer multiple of 2π. On the same principle, first term also can be ignored when we choose $P = 2\pi/k_{x0}$. For two plane waves, P1 and P2 can be chosen for their frequencies and incident angles. By choosing P as the least common multiple of P1 and P2, the sin-cos method can support two plane waves with different frequencies. Theoretically, proposed method can support infinite number of single-frequency plane waves. However, as the number of plane waves increases, P increases and efficiency of simulation decreases. In that cases, proposed method is not recommended.