## Maximum Endfire Directivity From a Continuous Line Source With a Uniform Phase Progression

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An optimized aperture distribution is investigated to achieve the maximum endfire directivity for a continuous line source with a uniform phase progression. In the optimization method, the aperture distribution is expanded as a set of cosine functions and the optimization reduces to solving a constrained linear-least-squares problem (IEEE Trans. AP, Oct. 2017, pp. 5123-5136).

For each assumed value of the phase constant  $\beta$  on the aperture, the numerical optimization procedure yields the magnitude of the aperture distribution that produces the maximum endfire directivity for that value of  $\beta$ . Results show that as the assumed value of  $\beta$  increases, so does the maximum obtainable directivity. Furthermore, as the value of  $\beta$  increases, the aperture distribution becomes more tightly concentrated near the center of the aperture.

For ordinary endfire ( $\beta = k_0$ ) the optimum aperture distribution is a uniform one. On the other hand, if ones assumes a uniform aperture distribution and allows  $\beta$  to be arbitrary, then the maximum endfire directivity occurs when the well-known Hansen Woodyard condition is met. If one starts with the value of  $\beta$  given by the Hansen Woodyard condition and then optimizes the aperture distribution, it is found that the optimum aperture distribution is not exactly uniform, but somewhat tapered at the aperture edges. As the value of  $\beta$  increases beyond the Hansen Woodyard condition, the optimum aperture distribution gets progressively more tapered at the aperture edges and thus becomes more concentrated at the center of the aperture.

Based on the copious computed results from the optimization, an approximate closed-form expression is derived for the maximum endfire directivity that is obtainable for a given aperture length  $L_a$  and assumed value of phase constant:

$$D^{\max} \approx \left(\frac{\pi L_a}{\lambda_0}\right)^2 \left(\frac{\beta}{k_0} - 1\right) + 4\frac{L_a}{\lambda_0},\tag{1}$$

where  $\lambda_0$  is the free-space wavenumber and  $k_0$  is the free-space wavenumber. The expression can serve as an upper bound for the endfire directivity for a line source with a uniform phase progression having a given length and value of  $\beta$ .

As seen from the above formula, the maximum directivity increases almost linearly with an increase in  $\beta$ . However, the radiated power decreases as  $\beta$  increases. Nevertheless, the enhancement in the directivity is much higher than that in previously studied cases of superdirectivity, under the same constraint in the ratio of reactive to radiated power. Also, the aperture distribution does not oscillate wildly as is does for superdirectivity, and is therefore more practical than for the usual cases of superdirectivity.