Design of Elliptical Shields for Generalized Source Integral Equations

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The Generalized Source Integral Equations (GSIE) is a family of integral formulations, designed to produce inherently compressible matrices, and thus enable fast direct solution. These formulations employ highly directional sources with complex radiation patterns rather than the conventional non-directional sources. Such sources suppress line-of-sight interactions between basis and testing functions on opposite sides of essentially convex scatterers, thus effectively reducing the problem's dimensionality. The dimensionality reduction enables compression and solution of two- and three-dimensional problems at $O(N\log N)$ and $O(N^{3/2})$ complexity, respectively, (N designates the number of unknowns in the discretized problem).

In an attempt to design such directional sources, we focus on sources that make use of elliptical scatterers as "shields". For these sources, a modified Green's function (MGF) comprising the direct free-space elemental source radiation and a contribution from the currents on the "elliptic shield" is computed. Having shields associated with each of the original geometry's basis functions, rather than a single large shield that is fixed in position and orientation, enables geometrical adaptivity and the extension of the method beyond essentially circular problems (Y. Brick, V. Lomakin, and A. Boag, "Fast Direct Solver for Essentially Convex Scatterers Using Multilevel Non-uniform Grids," IEEE Trans. Antennas and Propagation, vol. 62, no. 8, pp. 4314-4324, Aug. 2014).

For the Green's function of a PEC or a dielectric elliptic cylinder we follow the standard procedure of separating the Helmholtz equation in elliptic coordinates. The field outside the PEC elliptic cylinder or the fields inside and outside the dielectric elliptic cylinder are each expressed by means of products of radial and angular Mathieu functions. Radial Mathieu functions are expanded by Bessel functions to ensure a rapid convergence while angular Mathieu functions are developed as harmonic series. The boundary value problem is solved by enforcing the continuity of the tangential field components on the elliptic cylinder, i.e., by equating angular Mathieu functions. Different from the case of a dielectric circular cylinder the angular functions in domains with different material properties are not orthogonal which has to be considered in case of the dielectric elliptic cylinder.

To enable fast direct solution at an $O(N\log N)$ cost, several bottlenecks should be removed. In order to avoid the costly direct computation of the MGF, the Non-uniform Grid (NG) approach is used. Due to the different field behavior in different observation domains, several different grids should be constructed in order to gain fast MGF computation. Furthermore, several adjustments in the compression algorithm should be done in order to support the precomputed MGF. The elliptical shields design will be presented and compared with the earlier proposed shields employing electric and magnetic currents (A. Sharshevsky, Y. Brick, and Amir Boag, "Generalized Source Integral Equations," ICEAA 2017, Verona, Italy, Sept. 2017). The GSIE based direct solver's performance and accuracy will be demonstrated.