

# Reconstruction of Signal Waveform Received on Sea Surface based on Kolmogorov Method

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**Abstract**—It is analyzed the reconstruction performance of time domain waveform received on the sea surface depending only on the data of frequency amplitude spectrum by using Kolmogorov method. The basic reconstruction theory is introduced briefly and the comparison between signal waveform before and after reconstruction is provided based on simulation together with feature selective verification rules.

**Keywords**—signal reconstruction; minimum phase; feature selective verification

## I. INTRODUCTION

At present, although the technology of receiving signal frequency domain information is relatively mature, the technology of directly receiving the time domain information is still insufficient. Sometimes, we expect to get the time-domain waveform of the signal from the frequency-domain amplitude information of the signal. Therefore, it is very necessary to study the reconstruction of the signal waveform by using only the signal frequency domain information.

Under the normal circumstances, the condition of that the signal generated inversion, delay or signal amplitude of the positive and negative exchanged can not change the amplitude information in the frequency domain. In other words, in the general case, the amplitude spectrum of the signal in the frequency domain does not contain the above-mentioned information. Therefore, reconstruction of the signal waveform can not consider delay, reversal and amplitude of positive and negative cases of the signal [1]. However, in some special cases, when the signal to be reconstructed is the minimum phase signal or the maximum phase signal, its frequency domain amplitude information contains all its information in the time domain which provides a reference waveform of reconstruction of the signal waveform. As long as the reference waveform generates delay, reversal or amplitude of the positive and negative exchanged, we can get the original signal waveform. However, one of the preconditions must be met before using the amplitude spectrum to recover the signal waveform. In this paper, a minimum phase signal with typical noise is analyzed and evaluated.

## II. RECONSTRUCTION THEORY

The signal expression used in this paper is

$$y(t) = E_0 \times (e^{-a(t-t_1)} - e^{-b(t-t_1)}) \quad 0 \leq t \leq t_1 \quad (1)$$

where  $E_0 = 50V$ ,  $a = 4 \times 10^8$ ,  $b = 5 \times 10^8$ ,  $t_1 = 2 \times 10^8 ns$ . It is constructed on the base of double exponential function radar signal waveform in the IEC standard[1] which is shown in Fig.1.

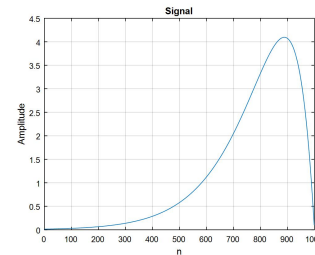


Fig.1. Signal waveform to be reconstructed

Before reconstructing the signal waveform, it is necessary to judge whether the signal satisfies the condition for recovering the time domain waveform from the amplitude spectrum. First, the discrete form of equation (1) is

$$y(n) = E_0 \times (e^{a(nT_s-t_1)} - e^{b(nT_s-t_1)}) \quad (2)$$

$$T_s = 1/f_0 \quad (3)$$

where  $f_0 = 5 \times 10^{10} Hz$ . Using Z transform for equation (3), we have

$$Y(Z) = E_0 \times \sum_{n=0}^N (e^{a(nT_s-t_1)} - e^{b(nT_s-t_1)}) \cdot Z^{-n} \quad (4)$$

It is obvious from (4) that all the poles of the signal are in the center of the unit circle and there is no zero point.

The prerequisite of reconstructing the signal waveform from the amplitude spectrum is that they belong to the group signals which has the same amplitude spectrum, namely, the theorem described as follow.

Set  $x(n)$  and  $y(n)$  is two sequences, the Z transformation does not contain a pole - zero reciprocal pairs; all its poles are in the unit circle except  $Z = \infty$ ; all its zero are not included in

the unit circle except  $Z=0$ . If amplitude spectrum of  $x(n)$  is equal to  $y(n)$ , then  $x(n) \sim y(n)$  [2].

We can see that the selected signal meets the prerequisite of reconstruction. Then the next step is to judge whether the selected signal is the minimum phase signal which is the sufficient condition for reconstruction and it is shown as follows [3].

$$(1) x = (x_0, x_1, \dots, x_n)$$

$$(2) X(Z) = x_0 + x_1 z^{-1} + \dots + x_n z^{-n}$$

(3) There is no zero point within the unit circle of  $X(Z)$ .

According to the formula (4), we know that the Z transform of  $x(n)$  is a finite sequence and there is no zero in the unit cell, so equation (4) is a minimum phase signal and the reconstructed signal is unique.

### III. SIMULATION RESULTS AND DISCUSSIONS

Since the selected signal  $x(n)$  is the minimum phase signal, so its amplitude spectrum can reconstruct the signal's waveform. In the MATLAB simulation, this paper selects the Kolmogorov method on the signal waveform reconstruction [4]. Although the Kolmogorov method of drawback is that when the zero near the unit circle, the error computed is relatively large. However, the signal used in this paper do not have zero near the unit circle. So using Kolmogorov method is reasonable.

The original signal and reconstructed signal waveform are shown in Fig.2 and the noise distribution is Rayleigh with the signal to noise ratio of 15dB.

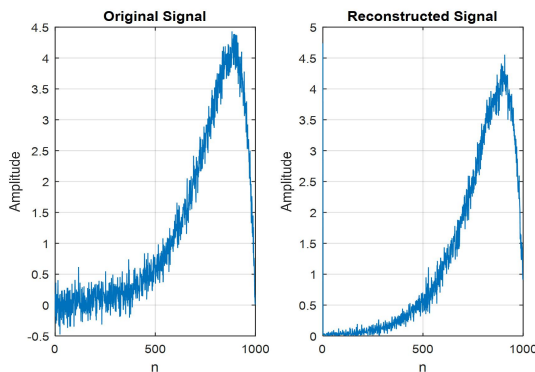


Fig.2. Original & reconstructed signal

According to the above ideas, We add Gaussian noise to the original signal before the signal waveform reconstruction. Then the method of Feature Selective Validation is used to

evaluate the fitting degree of the signal waveform reconstruction[5]. The result of evaluation is shown in Fig.3.

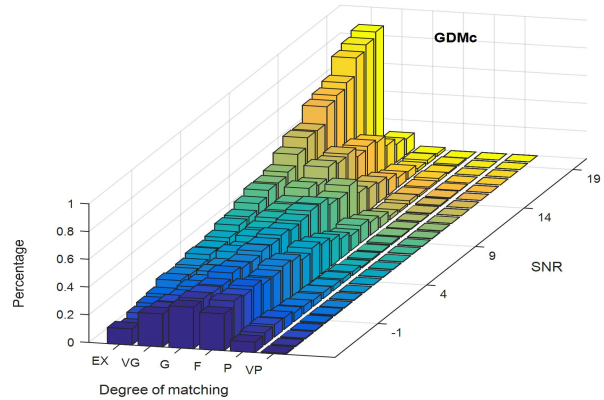


Fig.3. Reconstruction performance evaluation

In Fig3. percentage is ratio of number of fitting points to the number of total points. Degree of matching described the level of fitness. SNR means signal to noise ratio. It can seen clearly that when the SNR drops below 12dB, the fitting degree of the signal waveform begins to deteriorate .

### IV. CONCLUSION

Received signal waveform is not easy to measure with common device in the electromagnetic environment near sea surface. Based on data field strength, Kolmogorov method can be adopted to reconstruct the time domain waveform through it only does make sense for the minimum phase signals. New solution will be investigated in the future work.

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