A One-sided IA Algorithm Based on Sum Rate Maximization for MIMO Cognitive Network

Yibing Li, Hengyu Xu, Xueying Diao, Qian Sun
College of Information and Communication Engineering,
Harbin Engineering University,
Harbin, China
Email: liyibing0920@126.com, xhy952006@hrbeu.edu.cn, 2251408482@qq.com, sunqian215@126.com

***Abstract*—This paper focuses on applying interference alignment (IA) to the cognitive radio network where secondary users (SUs) transmit their information on the same frequency band of primary users (PUs) without generating interference on the primary network. We propose an IA scheme which runs at the transmitters only. The presented scheme considers the different interfering subspaces distances, and the distances between the interfering subspaces and the desired subspace. With the aim of optimizing the information rate of the network and guaranteeing user fairness at the same time, the proposed scheme moves the precoder obtained by the subspace distance optimization along the direction given by the gradient of the sum of logarithmic user data rate. Simulation shows the effectiveness of the proposed scheme in terms of the average sum rate.**

Keywords—cognitive radio; interference alignment; subspace distance; sum rate

I. INTRODUCTION

It has been a challenging problem to deal with the interference in the cognitive radio (CR) network. Recently, effective interference management scheme called interference alignment (IA) has been exploited to solve the interference problem in CR network [1]-[3]. [1] presented an improved version of minimum weighted leakage interference (IMWLI) scheme with a much faster convergence rate. In [2], an IA algorithm based on hierarchical interference suppression was proposed. In this paper, we propose a one-sided IA algorithm based on sum rate maximization (OIA-MSR). Compared with the traditional scheme that minimizing the global interference subspace distances, the OIA-MSR scheme has less spatial distances to be optimized with the number of users increasing, and considers the distances between the interfering subspaces and the desired subspace. With the aim of optimizing the information rate of the CR network and guaranteeing user fairness at the same time, the proposed scheme moves the precoder obtained by the subspace distance optimization along the direction given by the gradient of the sum of logarithmic user data rate. Simulation shows the effectiveness of OIA-MSR in terms of the average sum rate.

II. SYSTEM MODEL

Consider a K -user MIMO-CR interference network, which consists of K_p PUs and K_s SUs. The number of PU's transmit and receive antennas is denoted as $M_{t,p}$ and $M_{r,p}$. $M_{t,s}$ and $M_{r,s}$ respectively represents the number of SU's transmit and receive antennas. $\mathbf{V}_k$ is the k -th primary precoding matrix and $\mathbf{U}_k$ is the decoding matrix at the k -th primary receiver. $\mathbf{F}_l$ is the precoding matrix at the l -th secondary transmitter, and $\mathbf{W}_l$ is the l -th decoding matrix. The received signal at the k -th PU and the l -th SU is respectively given by

$$\mathbf{y}_k^p = \mathbf{U}_k^H \mathbf{H}_{kk} \mathbf{V}_k \mathbf{x}_k^p + \mathbf{U}_k^H \sum_{j=1, j\neq k}^{K_p} \mathbf{H}_{kj} \mathbf{V}_j \mathbf{x}_j^p + \mathbf{U}_k^H \sum_{l=K_p+1}^{K} \mathbf{H}_{kl} \mathbf{F}_l \mathbf{x}_l^s + \mathbf{U}_k^H \mathbf{n}_k^p \quad (1)$$

$$\mathbf{y}_l^s = \mathbf{W}_l^H \mathbf{H}_{ll} \mathbf{F}_l \mathbf{x}_l^s + \mathbf{W}_l^H \sum_{i=K_p+1, i\neq k}^{K} \mathbf{H}_{li} \mathbf{F}_i \mathbf{x}_i^s + \mathbf{W}_l^H \sum_{k=1}^{K_p} \mathbf{H}_{lk} \mathbf{V}_k \mathbf{x}_k^p + \mathbf{W}_l^H \mathbf{n}_l^s \quad (2)$$

$\mathbf{x}_k^p \in \mathbb{C}^{d_p \times 1}$ and $\mathbf{x}_l^s \in \mathbb{C}^{d_s \times 1}$ represent the transmitted signals. Each PU and SU respectively sends d_p and d_s independent data streams. $\mathbf{U}_k^H \mathbf{n}_k^p$ and $\mathbf{W}_l^H \mathbf{n}_l^s$ are the zero mean unit variance circularly symmetric additive white Gaussian noise vector. $\mathbf{H}_{kj}$ is the channel coefficient between the j -th transmitter and the k -th receiver. We define $\mathbf{P}_l$ as the power allocation of the l -th SU.

III. PROPOSED ALGORITHM

In this paper, we first introduce the concept of the aligned subspace and reduce the interference dimensions via aligning the subspaces spanned by interference with the aligned subspace. Since the interfering subspace cannot be guaranteed to be "far from" the desired subspace, resulting in a significant degradation of the signal-to-interference-plus-noise ratio (SINR), OIA-MSR adds the distances between the interfering subspaces and the desired subspace into the interference subspace distance minimization. We define $\mathbf{A}_i^s$ is the i -th secondary aligned matrix, whose columns span the i -th aligned subspace. According to [3], the distance between subspace $\mathbf{M}$ and subspace $\mathbf{N}$ can be expressed as $\left\| \mathbf{M} - \mathbf{N}\mathbf{N}^H \mathbf{M} \right\|_F^2$. Thus, the objective function of OIA-MSR is

$$\sum_{i=K_p+1}^{K} \sum_{l=K_p+1, l\neq i}^{K} \left\| \mathbf{H}_{il}\mathbf{F}_l - \mathbf{A}_i^s (\mathbf{A}_i^s)^H \mathbf{H}_{il}\mathbf{F}_l \right\|_F^2 - \left\| \mathbf{H}_{ii}\mathbf{F}_i - \mathbf{A}_i^s (\mathbf{A}_i^s)^H \mathbf{H}_{ii}\mathbf{F}_i \right\|_F^2$$
$$= \sum_{i=K_p+1}^{K} \sum_{l=K_p+1, l\neq i}^{K} \left\| (\mathbf{A}_i^s)^\perp [(\mathbf{A}_i^s)^\perp]^H \mathbf{H}_{il}\mathbf{F}_l \right\|_F^2 - \left\| (\mathbf{A}_i^s)^\perp [(\mathbf{A}_i^s)^\perp]^H \mathbf{H}_{ii}\mathbf{F}_i \right\|_F^2$$

According to the matrix theory, the optimal solution to the objective function is

$$(\mathbf{A}_i^s)^\perp = v_{M_{r,s}-d_s} \{ \sum_{l=K_p+1, l\neq i}^{K} \mathbf{H}_{il}\mathbf{F}_l (\mathbf{H}_{il}\mathbf{F}_l)^H - \mathbf{H}_{ii}\mathbf{F}_i (\mathbf{H}_{ii}\mathbf{F}_i)^H \} \quad (3)$$

where $v_d\{\bullet\}$ is the d least significant eigenvectors of a matrix.

The IA transmission strategy of the secondary network should consider not only the interference within the secondary user network, but also the cross-tier interference between primary and secondary users. To preserve PUs from the interference imposed by SUs, this paper resorts to a Frobenius norm constraint to eliminate the cross-tier interference. The optimization problem to calculate $\mathbf{F}_l$ is

$$\begin{aligned}&\min_{\mathbf{F}_l} \sum\nolimits_{i=K_p+1, i\neq l}^{K} \left\| (\mathbf{A}_i^s)^{\perp}[(\mathbf{A}_i^s)^{\perp}]^H \mathbf{H}_{il}\mathbf{F}_l \right\|_F^2 - \left\| (\mathbf{A}_l^s)^{\perp}[(\mathbf{A}_l^s)^{\perp}]^H \mathbf{H}_{ll}\mathbf{F}_l \right\|_F^2 \\ &s.t.: \left\| \mathbf{U}_k^H \mathbf{H}_{kl}\mathbf{F}_l \right\|_F = 0 \quad k = 1,\cdots,K_p\end{aligned} \tag{4}$$

It is not difficult to prove that the same solution in (4) is obtained for any unitary $\mathbf{Q}_l$ by replacing $\mathbf{F}_l$ by $\mathbf{F}_l\mathbf{Q}_l$, that is, the optimization problem in (4) can be extended to the Grassmann manifold. So the solution of the optimization problem is a set. Based on this character, within a class of equivalent solutions in (4), we can further seek a precoder that achieves the maximum data rate. With the aim of guaranteeing user fairness, we optimize the maximization of the sum of logarithmic user data rate as shown in (5) instead of directly optimizing the sum rate.

$$\max f(\mathbf{F}_l, l = K_p+1,\cdots K) = \sum\nolimits_{l=K_p+1}^{K} \ln R_l \tag{5}$$

where $R_l = \log_2\{\det[\mathbf{I}_{M_{r,s}} + \mathbf{\Psi}_l^{-1}(\mathbf{H}_{ll}\mathbf{F}_l\mathbf{P}_l\mathbf{F}_l^H\mathbf{H}_{ll}^H)]\}$. The conjugate gradient of (5) with respect to $(\mathbf{F}_l)^*$ is:

$$\nabla_{(\mathbf{F}_l)^*} f = \frac{1}{\ln 2}\sum_{i=K_p+1}^{K}\frac{1}{R_i}\mathbf{H}_{il}^H\mathbf{\Phi}_i^{-1}\mathbf{H}_{il}\mathbf{F}_l\mathbf{P}_l - \frac{1}{\ln 2}\sum_{i=K_p+1, i\neq l}^{K}\frac{1}{R_i}\mathbf{H}_{il}^H\mathbf{\Psi}_i^{-1}\mathbf{H}_{il}\mathbf{F}_l\mathbf{P}_l$$

where

$$\mathbf{\Phi}_l = \sigma^2\mathbf{I}_{M_{r,s}} + \sum\nolimits_{i=K_p+1}^{K}\mathbf{H}_{li}\mathbf{F}_i\,\mathbf{P}_i\mathbf{F}_i^H\mathbf{H}_{li}^H$$

$$\mathbf{\Psi}_l = \sigma^2\mathbf{I}_{M_{r,s}} + \sum\nolimits_{i=K_p+1, i\neq l}^{K}\mathbf{H}_{li}\mathbf{F}_i\,\mathbf{P}_i\mathbf{F}_i^H\mathbf{H}_{li}^H$$

Therefore, the optimal precoder can be achieved by moving the $\mathbf{F}_l$ obtained at each iteration of (4) in the direction of the gradient of (5) with respect to $(\mathbf{F}_l)^*$. The logical flow of the OIA-MSR algorithm in the secondary network is given as follows:

1. Calculate $(\mathbf{A}_i^s)^{\perp}, i = K_p+1,\cdots,K$ according to (3);
2. Perform (4) to acquire $\mathbf{F}_l, l = K_p+1,\cdots,K$;
3. Compute the tangent vector $\mathbf{Z}_l = (\mathbf{I}_{M_{t,s}} - \mathbf{F}_l(\mathbf{F}_l)^H)\nabla_{\mathbf{F}_l^*} f$;
4. Move the variable $\mathbf{F}_l, l = K_p+1,\cdots,K$ along the iteration trajectory $\mathbf{F}_l = (\mathbf{F}_l\mathbf{N}_l(\cos(\mathbf{\Lambda}_l t))\mathbf{N}_l^H) + (\mathbf{M}_l(\sin(\mathbf{\Lambda}_l t))\mathbf{N}_l^H)$ to update $\mathbf{F}_l$, where $\mathbf{M}_l$ and $\mathbf{N}_l$ denote the left and right singular vectors of $\mathbf{Z}_l$ respectively. $\mathbf{\Lambda}_l$ is the singular matrix.
5. Continue until convergence.
6. The decoding matrix is designed by max-SINR [4].

IV. SIMULATION RESULTS AND ANALYSIS

In this section, we provide some simulation examples to evaluate the performance of the proposed algorithms. The step size t in the proposed scheme is 0.3. Simulation results are averaged over 500 channels. The noise power is normalized to one. The primary and secondary system is respectively represented by $(M_{t,p}, M_{r,p}, d_p)$ and $(M_{t,s}, M_{r,s}, d_s)$. For the primary network, the OIA-MSR algorithm is also applicable, but there is no need to consider cross-tier interference when designing the precoding matrix.

Fig. 1 shows the average total sum rate of the primary and secondary links, where a four-user primary system and a three-user secondary system is considered. Obviously, the achievable sum rate of the proposed scheme is higher than that of other algorithms. According to [2], it is clear that all the SU can only eliminate the interference imposed by PUs, thus, the interference among SUs lead to poor performance of the secondary system. In Fig. 2, a two-user primary system and a four-user secondary system is considered. Similarly, due to the same reason, the achievable sum rate of the algorithm based on hierarchical interference suppression is lower than that of OIA-MSR. Since IWMLI ignores the influence of noise and makes no attempt to maximize the desired signal power within the desired signal subspace, the sum rate achieved by IMWLI is the worst.

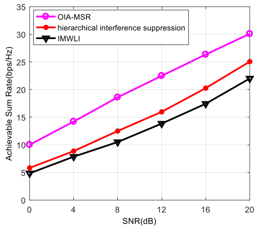

Fig. 1. Sum rate of the system versus SNR, where the PU and the SU is equipped with $(4,2,1)$ and $(6,6,2)$

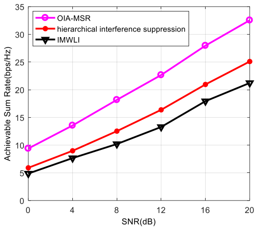

Fig. 2. Sum rate of the system versus SNR, where the PU and the SU is equipped with $(4,4,2)$ and $(6,6,2)$

V. CONCLUSION

In this paper, we propose a one-sided IA algorithm, which considers the different interfering subspaces distances, and the distances between the interfering subspaces and the desired subspace. With the aim of optimizing the information rate of the CR network and guaranteeing user fairness at the same time, the OIA-MSR scheme moves the precoder along the direction given by the gradient of the sum of logarithmic user data rate. Simulations show that the OIA-MSR scheme have obvious improvement in terms of the average sum rate.

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