A Two-dimensional DOA Estimation Method for very Low SNR

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Abstract—In this paper, we propose a new method to mitigate the effect of low signal to noise ratio (SNR) on two-dimensional (2-D) Direction of arrival (DOA) estimation. This method consists to extend the antenna steering vectors before applying the MUltiple SIgnal Classification (MUSIC) algorithm for 2-D DOA estimation. The simulation results show a good location accuracy enhancement.

Keywords—2-D DOA estimation; SNR; steering vectors; MUSIC

I. Introduction

Two-Dimensional direction of arrival (2-D DOA) estimation is very important in wireless communication systems [1]. MUltiple SIgnal Classification (MUSIC) algorithm is the most popular algorithm for DOA estimation, and L-shaped array is the most planar array used because of his larger array aperture [2], and his low computational complexity. A sparse L-shaped array is used with propagator method [3], when an ear block array is used in [4] to make DOA estimation, however, the signal to noise ratio (SNR) is a main parameter which affect the performances of these methods.

In this paper we propose a method to mitigate the effect of the noise on the DAO estimation.

This paper is organised as follows: section II and III describes the signal model, and introduce briefly the MUSIC algorithm. The proposed method is explained in section IV. Section V shows the simulation results and the section VI conclude the paper.

II. SIGNAL MODEL

We suppose K uncorrelated far-field narrowband signals with same wavelength λ . The angles of arrival of signals are (θ_k, ϕ_k) where θ_k and ϕ_k denote respectively the elevation and azimuth angles of kth incident signal, and k = 1, ..., K. The L-shaped array is shown in Fig. 1. We use same L-shaped array as in [5]. Both Uniform Linear Array (ULAs) are identical and each one contains M antennas, with a common element at the origin, and the elements spacing is $d = \lambda/2$. The incidents signals on x-axis and y-axis are denoted as:

$$X(t) = A_x.S(t) + N_x(t)$$
 (1)

$$Y(t) = A_{\nu}.S(t) + N_{\nu}(t) \tag{2}$$

 $A_r = [a_{r1}, \dots, a_{rK}]$ Where: (3)

$$A_{v} = \left[a_{v,1}, \dots, a_{v,K} \right] \tag{4}$$

$$a_{x,k} = \left[1, e^{j\varphi_{x,k}}, \dots, e^{j\varphi^M}_{x,k}\right]^T \tag{5}$$

$$a_{y,k} = \left[1, e^{j\varphi_{y,k}}, ..., e^{j\varphi^{M}}_{y,k}\right]^{T}$$
 (6)

$$\varphi^{m}_{x,k} = 2\pi \cdot \frac{d}{\lambda} \cdot (m-1) \cdot \sin \theta_{k} \cdot \cos \phi_{k} \tag{7}$$

$$\varphi^{m}_{y,k} = 2\pi \cdot \frac{d}{\lambda} \cdot (m-1) \cdot \sin \theta_{k} \cdot \sin \phi_{k}$$
 (8)

 A_x and A_y denote the steering vectors matrices and $a_{x,k}$ and $a_{v,k}$ denote the steering vector for the kth incident signal.

 $N_x(t)$ and $N_y(t)$ are gaussian white noise matrices and S(t) is the signals matrix.

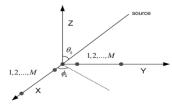


Fig. 1. The geometrical configuration of the L-shaped array

The covariances matrices are estimated by:

$$\hat{R}_{xx} = \frac{1}{L} . X . X^H \text{ and } \hat{R}_{yy} = \frac{1}{L} . Y . Y^H$$
 (9)

Where L denotes the number of sample samples snapshots

III. MUSIC CLASSIC

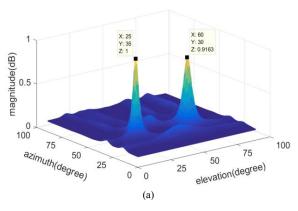
The principle is to project the steering vectors on the noise subspace. After that, we get the following function: $F(\phi,\theta) = a(\phi,\theta)^{H}.E_{n}.E_{n}^{H}.a(\phi,\theta)$

$$F(\phi, \theta) = a(\phi, \theta)^{H} \cdot E_{n} \cdot E_{n}^{H} \cdot a(\phi, \theta)$$
 (10)

Where $[]^H$ denote the Hermitian.

The zeros of this function maximize the following spectral function, and are the directions of arrival of the incidents signals.

$$P_{MUSIC} = \frac{1}{a(\phi,\theta)^H \cdot E_n \cdot E_n^H \cdot a(\phi,\theta)}$$
(11)



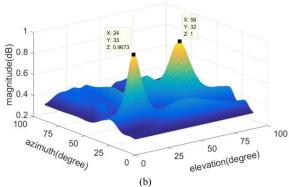


Fig. 2. 2-D MUSIC spectrum at SNR = -13 dB With the proposed method in (a) and the method [6] in (b).

IV. PROPOSE METHOD

The proposed method consists to transform the steering vector before applied the DOA estimation algorithm. First, we subdivide a_{xk} into two vectors, a_1 and a_2 , such as

$$a_1 = \left[1, e^{j\varphi_{x,k}}, \dots, e^{j\varphi^{M-1}_{x,k}}\right]^T$$
 (12)

$$a_2 = \left[e^{j\varphi_{x,k}}, \dots, e^{j\varphi_{x,k}^M} \right]^T \tag{13}$$

After that we obtain a third vector a_3 such as:

$$a_3 = a_2 \mathbf{Q} \ a_2 \tag{14}$$

Where " \bigcirc " denote the Hadamard product.

Finally, we make a vertical concatenation of a_1 , a_2 and a_3 to get a_4 , the new steering vector.

Then,
$$a_4 = [a_1, a_2, a_3]^T$$

The same procedure is applied to a_{yk} .

After that the covariance matrix is estimated and MUSIC algorithm is applied.

V. SIMULATION RESULTS

In this section, we present some simulation results to compare the propose algorithm with that in [5]. we considered two uncorrelated signals source positioned at $(25^{\circ}, 35^{\circ})$ and $(60^{\circ}, 30^{\circ})$. The antennas number of both ULAs is M = 10, with inter sensors spacing $d = \lambda/2$. The number of samples snapshot is 500. The fig. 2. Shows the DOAs estimated spectrums of the proposed method in (a), and the method in [5] in (b). SNR is set at -13dB. The spectrum in (b) is deformed because of the low SNR, and there are errors in DOAs estimations. The reals DOAs are $(25^{\circ}, 35^{\circ})$ and $(60^{\circ}, 30^{\circ})$ but the estimated are $(24^{\circ}, 33^{\circ})$ and $(58^{\circ}, 32^{\circ})$. In (a) there are no spectrum deformation and no DOA estimation errors.

In the figure 3 The Root Mean Square Error (RMSE) of the propose method his compared with that of the method in [5] the results show that the proposed method provides better performances than the method in [5] for the low SNR.

$$RMSE = \sqrt{E\left[\left(\hat{\theta} - \theta\right)^2 + \left(\hat{\phi} - \phi\right)^2\right]}$$
 (15)

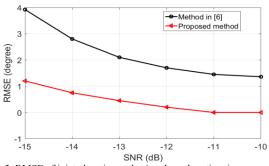


Fig. 3. RMSE of joint elevation and azimuth angle estimations versus SNR for both uncorrelated sources

VI. CONCLUSION

In this paper, we proposed a new method for DOA estimation of uncorrelated signals, when SNR is very low. The simulation results show the high performance of the proposed approach at SNR lower than -10dB compared to method in [5]. As perspective work, our approach deserves to be used with other algorithms such as ESPRIT or ROOT-MUSIC.

REFERENCES

- [1] N. Tayem, "2D DOA estimation of multiple coherent sources using a new antenna array configuration," in 2012 Conference Record of the Forty Sixth Asilomar Conference on Signals, Systems and Computers (ASILOMAR), 2012, pp. 212-216.
- [2] Y. Yuxuan, Z. Zhi, Y. Haifen, Y. Jiao, and G. Yan, "A novel 2-D DOA estimation method via sparse L-shaped array," in 2016 2nd IEEE Inter. Conf. on Computer and Communications (ICCC), 2016, pp. 1865-1869.
- [3] N. Tayem and H. M. Kwon, "L-shape 2-dimensional arrival angle estimation with propagator method," IEEE Transactions on Antennas and Propagation, vol. 53, pp. 1622-1630, 2005.
- [4] S. O. Al-Jazzar, H. J. Strangeways, and D. C. McLernon, "2-D angle of arrival estimation using a one-dimensional antenna array," in 2014 22nd European Signal Processing Conference (EUSIPCO), 2014, pp. 1905-1909.
- [5] Z. Xiaofei, L. Jianfeng, and X. Lingyun, "Novel two-dimensional DOA estimation with L-shaped array," EURASIP Journal on Advances in Signal Processing, vol. 2011, p. 50, 2011/08/30 2011.